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Extension of TOPSIS model for multi-criteria decision making

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***Abstract*— Multi criteria group decision making methods are broadly used in the real-world decision circumstances for homogeneous groups. Decision-making problems often involve a complex decision-making process in which multiple requirements and uncertain conditions have to be taken into consideration simultaneously. In this paper, we consider the ideal solution and the anti-ideal solution and assess each alternative in terms of similarity to the ideal solution and the anti-ideal solution. To minimize the error, the normalization of fuzzy data is carefully avoided. To get greater accuracy in ranking fuzzy rating, we use the latest and advanced similarity measure. The proposed method is more flexible in modeling the decision makers preferences and more appropriate and effective to handle multi-criteria problems of considerable complexity.**

Keywords*— Trapezoidal fuzzy number, Multi criteria group decision making, Similarity measure.

I. INTRODUCTION

In 1981, C. L. Hwang and K. Yoon [13] first developed a **T**echnique for **O**rder **P**erformance by **S**imilarity to the **I**deal **S**olution (TOPSIS) for solving **M**ulti-**C**riteria **D**ecision-**M**aking (MCDM) problems. It helps decision maker(s)(DMs) organize the problems to be solved, and carry out analysis, comparisons and rankings of the alternatives. The basic principle of the TOPSIS is that chosen alternative should have the largest ideal solution from positive ideal solution and least ideal solution from negative ideal solution. In classical methods for multi-criteria decision making problems, the ratings and weights of criteria are known precisely. In the classical TOPSIS method, the ratings of alternatives and the weights of criteria are presented by real values, too. The classical TOPSIS method has been successfully used in different fields [5], [19].

In the past few years, numerous attempts to handle this vagueness, imprecision, and subjectiveness have been carried out to apply fuzzy set theory to multiple criteria evaluation methods [1, 2, 5, 24, 25, 26]. The overall utility of the alternatives with respect to all criteria is often represented by a fuzzy number, which is named the fuzzy utility and is often referred to by fuzzy multi-criteria evaluation methods. The ranking of the alternatives is based on the comparison of their corresponding fuzzy utilities [3, 5, 27, 14]. Multi-criteria evaluation methods are used widely in fields such as information project selection [14, 15], material selection [19], and many other areas of management decision problems [19] and strategy selection problems [4, 7, 9, 21]. Tsaur et al.[20] first convert a fuzzy MCDM problem into a crisp one via centroid defuzzification and then solve the non-fuzzy MCDM problem using the TOPSIS. Hsu and Chen [12] discuss an aggregation of fuzzy opinions under group decision making. Li [18] proposes a simple and efficient fuzzy model to deal with multi-judges/MCDM problems in a fuzzy environment. Liang [17] incorporates fuzzy set theory and the basic concepts of positive ideal and negative ideal points and extends MCDM to a fuzzy environment.

In 2003 Chen.S.J and Chen.S.M.[6] introduced a similarity measure using trapezoidal fuzzy numbers. Hejazi et al.[11] also introduced similarity measure between two trapezoidal fuzzy numbers. In 2010 Xu et al.[23] initiated new similarity measure of trapezoidal fuzzy numbers.

P.Dheena and G.Mohanraj [8] proposes the ideal solution and anti-ideal solution and assess each alternative in terms of distance as well as similarity to the ideal solution and anti-ideal solution.

This paper is organized as follows: The extension of TOPSIS for fuzzy multi-criteria decision making in section3.In section 4; an illustrative numerical example is given to apply the fuzzy multi-criteria method for alternatives of evaluating university faculty for tenure and promotion.

II. PRELIMINARIES

Definition *2.1.* *Let a_1, a_2, a_3, a_4 be any real numbers, such that $-\infty<a_1\le a_2\le a_3\le a_4\le\infty$. The membership function of a trapezoidal fuzzy number $\hat{T}$ has of the form is given by*

$$\mu_{\hat{T}}(x)=\begin{cases}0 & if\ x<a_1\\ \dfrac{x-a_1}{a_2-a_1} & if\ a_1\le x\le a_2\\ 1 & if\ a_2\le x\le a_3\\ \dfrac{a_4-x}{a_4-a_3} & if\ a_3<x\le a_4\\ 0 & if\ x>a_4\end{cases}$$

Definition *2.2.* *Let a_1,a_2,a_3,a_4 be any real numbers, such that $-\infty<a_1\le a_2\le a_3\le a_4\le\infty$ and let w be the weight such that $0\le w\le 1$. Then the membership function is given by*

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$$\mu_{\hat{T}}(x)=\begin{cases}0 & if\ x<a_1\\ w\times\left(\dfrac{x-a_1}{a_2-a_1}\right) & if\ a_1\le x\le a_2\\ w & if\ a_2\le x\le a_3\\ w\times\left(\dfrac{a_4-x}{a_4-a_3}\right) & if\ a_3<x\le a_4\\ 0 & if\ x>a_4\end{cases}$$

Similarity measure: *Let* $\hat{A}=(a_1,a_2,a_3,a_4;w_A)$ *and* $\hat{B}=(b_1,b_2,b_3,b_4;w_B)$ *be two generalized trapezoidal fuzzy numbers Xu et al.[23] proposed the similarity measure* $S(\hat{A},\hat{B})$ *that is given by*

$$S(\hat{A},\hat{B})=1-\frac{1}{8}\sum_{i=1}^{4}|a_i-b_i|-\frac{d(\hat{A},\hat{B})}{2} \qquad (2.1)$$

where

$$d(\hat{A},\hat{B})=\sqrt{\frac{(x^*_{\hat{A}}-x^*_{\hat{B}})^2+(y^*_{\hat{A}}-y^*_{\hat{B}})^2}{1.25}}$$

$$y^*_{\hat{A}}=\begin{cases}w_{\hat{A}}\times\dfrac{\left(\dfrac{a_3-a_2}{a_4-a_1}+2\right)}{6} & if\ a_4\ne a_1\\ \dfrac{w_{\hat{A}}}{2} & if\ a_4=a_1\end{cases}$$

$$y^*_{\hat{B}}=\begin{cases}w_{\hat{B}}\times\dfrac{\left(\dfrac{b_3-b_2}{b_4-b_1}+2\right)}{6} & if\ b_4\ne b_1\\ \dfrac{w_{\hat{B}}}{2} & if\ b_4=b_1\end{cases}$$

$$x^*_{\hat{A}}=\begin{cases}\dfrac{y^*_{\hat{A}}\times(a_3+a_2)+(a_4+a_1)\times(w_{\hat{A}}-y^*_{\hat{A}})}{2w_{\hat{A}}}, & if\ w_{\hat{A}}\ne 0;\\ \dfrac{a_4+a_1}{2}\ , & if\ w_{\hat{A}}=0\end{cases}$$

$$x^*_{\hat{B}}=\begin{cases}\dfrac{y^*_{\hat{B}}\times(b_3+b_2)+(b_4+b_1)\times(w_{\hat{B}}-y^*_{\hat{B}})}{2w_{\hat{A}}}, & if\ w_{\hat{B}}\ne 0;\\ \dfrac{a_4+a_1}{2}\ , & if\ w_{\mathbf{B}}=0\end{cases}$$

Linguistic variables: *Let* $L^u=(l^u_0,l^u_1,l^u_2,...,l^u_t)$ *be the* u^{th} *pre-established finite and totally ordered linguistic term set* $t=1,2,3,...,t+1$, *where* l^u_i *be the i-th linguistic term of* L^u *and* $t+1$ *is the cardinality of* L^u, l^u_i *can be approximately expressed as a trapezoidal fuzzy number.The* i^{th} *linguistic variables* l^u_i *is expressed as* $\hat{d}^u_i$ *by a formula[8] given by* $\hat{d}^u_i=(d^{u_1}_i,d^{u_2}_i,d^{u_3}_i,d^{u_4}_i)$

$$=\left(\max\left\{\frac{2i-1}{2t+1},0\right\},\frac{2i}{2t+1},\frac{2i+1}{2t+1},\min\left\{\frac{2i+2}{2t+1},1\right\}\right)$$

(2.2)

TABLE 3.1. *The raetings of the three alternatives by decision expertsunder all criteria*

Decision makers

Criteria	*Alternatives*	*D₁*	*D₂*	*D₃*
C₁:Teaching	*A₁*	*MP*	*F*	*F*
	A₂	*F*	*G*	*F*
	A₃	*MP*	*G*	*MG*
	A₄	*VG*	*GF*	*VG*
	A₅	*MG*	*MG*	*MG*
C₂:Research	*A₁*	*G*	*MP*	*VG*
	A₂	*VG*	*F*	*MP*
	A₃	*MP*	*G*	*G*
	A₄	*MG*	*MG*	*MG*
	A₅	*G*	*VG*	*G*
C₃:Service	*A₁*	*G*	*G*	*VG*
	A₂	*VG*	*VG*	*G*
	A₃	*VG*	*MG*	*F*
	A₄	*MP*	*MG*	*VG*
	A₅	*MG*	*F*	*F*

III. EXTENSION OF TOPSIS FOR FUZZY MULTI-CRITERIA DECISION MAKING

A multi-criteria decision making problem is to select best alternatives from the set of alternatives by consisting set of criteria. The classical TOPSIS method is based on the idea that the best alternative have the largest similarity to the positive ideal solution and least similarity of the negative ideal solution. The positive ideal solution is compose of the best achievable values of local criteria while a negative ideal solution least achievable values of the local criteria.

Suppose multi-criteria decision making problem based on "m" –alternatives and "n"-criteria. There are "k" decision makers now give their ratings and alternative with respect to criteria.

The TOPSIS method consists of the following steps:

Step 1: The set of linguistic variable is given by the Table 3.2 Let DM_1, DM_2,… ,DM_k, be the k -set of decision makers. Let $l_{ij}^{(u)}$ be the linguistic variable is given by u^{th} decision maker DM_u to the i^{th} alternative A_i with respect to criteria C_j , which is give by the Table 3.1

TABLE 3.2: *LINGUISTIC TERMS AND CORRESPONDING TRAPEZOIDAL FUZZY NUMBERS.*

Linguistic terms	Trapezoidal fuzzy number
Low	(0,0,0.0769,0.1538)
Medium Low	(0.0769,0.1538,0.2308,0.3077)
Medium	(0.2308,0.3077,0.3846,0.4615)
Medium High	(0.3846,0.4615,0.5385,0.6154)
High	(0.5385,0.6154,0.6923,0.7692)
Very High	(0.6923,0.7692,0.8462,0.9231)
Extremely High	(0.8462,0.9231,1,1)

Step 2: The linguistic variables is converted into fuzzy trapezoidal numbers by the Equation 2.2.

Step 3: Let $\hat{r}_{ij}^{u}$ be the fuzzy trapezoidal number which is converted by the Equation 2.2 , corresponding to the linguistic variable $l_{ij}^{(u)}$.

Step 4: The normalized decision matrix

$$\hat{D} = \begin{pmatrix} \hat{v}_{11} & \hat{v}_{12} & . & . & . & \hat{v}_{1n} \\ \hat{v}_{21} & \hat{v}_{22} & . & . & . & \hat{v}_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \hat{v}_{m1} & \hat{v}_{m2} & . & . & . & \hat{v}_{mn} \end{pmatrix}$$

is calculated as follows

$$\hat{v}_{ij} = \sum_{u=1}^{k} r_{ij}^{u} w^{u} \qquad \textbf{(3.3)}$$

where $w^1, w^2, w^3, ..., w^k$ is the weight of DM_1, DM_2,…, DM_k.

Step 5: Let B be set of the benefit criteria and C be the cost criteria. The positive ideal solution A_j^+ is given by

$$A_j^+ = \begin{cases} (1,1,1,1;1) & if\ i \in B \\ (0,0,0,0;1) & if\ i \in C \end{cases}$$

For all i=1,2,3,…n.

The negative ideal solution A_j^- is given as follows:

$$A_j^- = \begin{cases} (0,0,0,0;1) & if\ i \in B \\ (1,\ 1,\ 1,\ 1;\ 1) & if\ i \in C \end{cases}$$

For all i=1,2,3,…,n.

Step 6: Let S_{ij}^+ and S_{ij}^- be the similarity measures of $S(\hat{v}_{ij}, A_j^+)$ and $S(\hat{v}_{ij}, A_j^-)$ respectively which are calculated by the Equation 2.1

Step 7: The rank of alternative $R(A_i)$ is calculated by the formula

$$\sum_{j=1}^{n} \left(\frac{S_{ij}^+ + 1 - S_{ij}^-}{2} \right) \times w_j \qquad \textbf{(3.4)}$$

Where $w_1, w_2, w_3, ..., w_n$ be the weights of $n-$criteria $(C_1, C_2, C_3, ... C_n)$.

Step 8: The top rank of alternative is selected to be the best alternative.

IV. NUMERICAL EXAMPLE

In this section, we work out a numerical example to illustrate the TOPSIS approach for decision making problem with fuzzy data. Here multi-criteria decision making problem of evaluating university faculty for tenure and promotion.

Five faculty candidates are the alternatives denoted by $A = \{A_1, A_2, A_3\ , A_4, A_5\}$.The criteria are used at university are C_1 : Teaching, C_2 : Research, C_3 : Service and the weight vector $w =$ (0.36; 0.31; 0.33) for criteria (C_1, C_2, C_3). The alternatives $A = \{A_1, A_2, A_3\ , A_4, A_5\}$ are evaluated using linguistic values by decision makers DM = DM_1, DM_2, DM_3 whose weight vector $\lambda =$ (0.4, 0.5, 0.1) under this criteria. Here all criteria is benefit criteria.

Step 1: Decision maker's rating in linguistic variable is given in the Table 3.2.

Step2: The linguistic variable converted into trapezoidal fuzzy numbers by the Equation 2.2.

Step 3: $\hat{v}_{ij}$ is calculated by the Equation 3.3.

Step 4: Normalized decision matrix is calculated by the Equation 3.3.

Step 5: Positive ideal solution $A_j^+ = (1, 1, 1, 1; 1)$ and negative ideal solution $A_j^- = (0, 0, 0, 0; 1)$.

Step 6: S_{ij}^+ and S_{ij}^- as calculated and tabulated in Table 4.1and 4.2

TABLE 4.1. *SIMILARITY MEASURE FOR POSITIVE IDEAL SOLUTION* S_{ij}^+

	A_1	A_2	A_3	A_4	A_5
C_1	0.284151852	0.474464158	0.434375989	0.559535826	0.474464158
C_2	0.4161558236	0.510643781	0.437234174	0.469298464	0.652830065
C_3	0.612113381	0.710296354	0.562097759	0.391129334	0.391127069

TABLE 4.2. *SIMILARITY MEASURE FOR NEGATIVE IDEAL SOLUTION* S_{ij}^-

	A_1	A_2	A_3	A_4	A_5
C_1	0.652462673	0.474464158	0.514020213	0.380529530	0.47464158
C_2	0.521447474	0.425388402	0.488704536	0.469298464	0.277831249
C_3	0.323034603	0.214650561	0.377677782	0.549442439	0.549439563

Step 7: Rank of each alternatives is calculated and tabulated in Table 5.5.

TABLE 4.3. *RANK OF EACH ALTERNATIVE*

A_i	$R(A_i)$
A_1	0.465081844
A_2	0.594996140
A_3	0.50811543
A_4	0.519145255
A_5	0.532003254

V. CONCLUSION

In this method, normalization is carefully avoided to minimize the error. The alternatives are rankers as $A_2 \succ A_5 \succ A_4 \succ A_3 \succ A_1$.The alternative A_2 is selected to be the best alternative.

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