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Equitable coloring of Sudha gird graohs and Sudha graph

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***Abstract*— An equitable coloring of a graph is an assignment of colors to the vertices of the graph, in such a way that no two adjacent vertices have the same color and the number of vertices in any two color classes differ by at most one. Sudha gave the construction of the following graphs:**

(i) Sudha gird of diamonds $S_d(m,n)$,
(ii) Sudha gird of hexagons $S_h(m,n)$ **and**
(iii) Sudha graph $S(n,m)$.

In this paper, we have discussed the equitable coloring of the Sudha gird of diamonds, Sudha gird of hexagons and Sudha graph.

Keywords*— *Sudha gird of diamonds; Sudha gird of hexagons; Sudha graph; Equitable coloring; Color class and Equitable chromatic number.

AMS Subject Classification : 05C15

I. INTRODUCTION

Mayer [1] introduced the equitable coloring of graphs, Hanna et. al. [2, 3, 4] have discussed about the complexity of equitable vertex coloring of graphs, equitable coloring of some graph products, corona product of graphs and cubic graphs. Hanna [5] also elaborately discussed about the equitable coloring of corona mutiproducts of graphs. Sudha et. al. [6] have discussed about the equitable coloring of prisms and the generalized Petersen graphs. Lih et. al. [7] found the equitable coloring of trees. Dorothee [8] gave the equitable coloring of complete multipartite graph. Sudha et. al. [9] introduced the Sudha graph $S(n,m)$ and found the total coloring of $S(n,m)$ graph.

In this paper, we have discussed the equitable coloring of the Sudha gird of diamonds $S_d(m,n)$, Sudha gird of hexagons $S_h(m,n)$ and $S(n,m)$ graphs. Moreover, we found that the equitable chromatic number of $S_d(m,n)$ is either 2 or 3, the equitable chromatic number of $S_h(m,n)$ is 2 and the equitable chromatic number of $S(m,n)$ is either 3 or 4.

Definition 1.1. *Vertex coloring is the coloring of the vertices of the graph with the minimum number of colors so that no two adjacent vertices have the same color.*

Definition 1.2. *The set of vertices having the same color in the vertex coloring of a graph are said to be in that color class.*

In k-coloring of a graph, there are k color classes. The color classes are represented by $C[1], C[2], \ldots,$ if $1, 2, \ldots$ represent the colors.

Definition 1.3. *A graph G is said to be equitable k-colorable if its vertex set V can be partitioned into k disjoint subsets $V_1, V_2, \ldots, V_k$, satisfy the condition $\big||V_i| - |V_j|\big| \le 1$ for all $1 \le i \le k, 1 \le j \le k$.*

The smallest integer k for which G is equitable k-coloring is known as the equitable chromatic number of G and is denoted by $\chi_= G$.

Definition 1.4. *Sudha grid of diamonds $S_d(m,n)$ is an induced subgraph of the tensor product of two paths P_m with the vertices $u_1, u_2, u_3, \ldots, u_m$ for odd $m \ge 3$ and P_n with the vertices $v_1, v_2, v_3, \ldots, v_n$ for odd $n \ge 3$ with the vertex set $V(S_d(m,n)) =$*

$$\left\{ u_i v_j \,/\, \begin{array}{c} \text{either } i \equiv 1 \ (mod\ 2) \text{ and } j \equiv 0 \ (mod\ 2) \\ \text{or } i \equiv 0 \ (mod\ 2) \text{ and } i \equiv 1 \ (mod\ 2) \end{array} \right\}$$

and the edge set

$$E(S_d(m,n)) = \{(u_i v_j)(u_i v_j)/u_i u_k \in E(P_m) \text{ and } v_j v_k \in E(P_n)\}$$

Instead of denoting the vertices as $u_i v_j$, we denoted them as $v_{i,j}$ for simplicity.

Illustration 1.5

$S_d(5,5)$ is a Sudha gird of diamonds with the vertex set $\{v_{1,2}, v_{1,4}, v_{2,1}, v_{2,3}, v_{2,5}, v_{3,2}, v_{3,4}, v_{4,1}, v_{4,3}, v_{4,5}, v_{5,2}, v_{5,4}\}$ as shown in figure 1.

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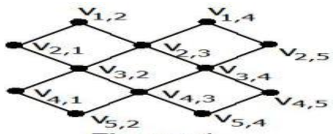


Figure 1

Definition 1.6. *Sudha grid of hexagons $S_h(m,n)$ is an induced subgraph of the strong product of two paths P_m with the vertices $u_1, u_2, u_3, \ldots, u_m$ for odd $m \geq 3$ and P_n with the vertices $v_1, v_2, v_3, \ldots, v_n$ for $n \equiv 0 \pmod 4$ with the vertex set*

$$V(S_h(m,n)) = \{u_i v_j / \, i+j \equiv 1 \pmod 2 \text{ and } i+j \equiv 0 \pmod 2\}$$

and the edge set

$$E(S_d(m,n)) = \{(u_i v_j)(u_i v_j)/u_i u_k \in E(P_m) \text{ and } v_j v_k \in E(P_n)\}.$$

Illustration 1.7

$S_h(8,7)$ is a Sudha gird of hexagons graph with the vertex set $\{v_{1,2}, v_{1,4}, v_{1,6}, v_{2,1}, v_{2,3}, v_{2,5}, v_{2,7}, v_{2,1}, v_{3,3}, v_{3,5}, v_{3,7}, v_{4,2}, v_{4,4}, v_{4,6}, v_{5,2}, v_{5,4}, v_{5,6}, v_{6,1}, v_{6,3}, v_{6,5}, v_{6,7}, v_{7,1}, v_{7,3}, v_{7,5}, v_{7,7}, v_{8,2}, v_{8,4}, v_{8,6}\}$ as shown in figure 2.

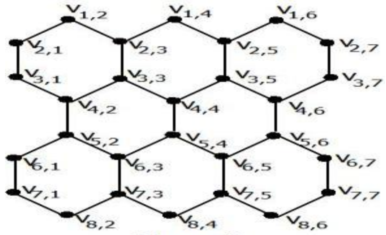


Figure 2

Definition 1.8. *Sudha graph denoted by $S(n,m)$ is defined as the graph with n vertices $\{v_i\}$, $1 \leq i \leq n$ and the following edges for $1 \leq i \leq n$*

(i) v_i is adjacent to $v_{i+1}$and v_n is adjacent to v_1
(ii) $v_i$is adjacent to v_{i+m} if $i+m < n$ (iii)
(iii) v_i is adjacent to v_{i+n-m} if $i+m \geq n$.

Illustration 1.9 (iv)

$S(9,2)$ is a Sudha graph with the vertex set $v_1, v_2, v_3, \ldots, v_9$ as shown in figure 3.

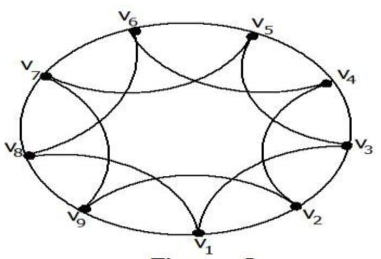
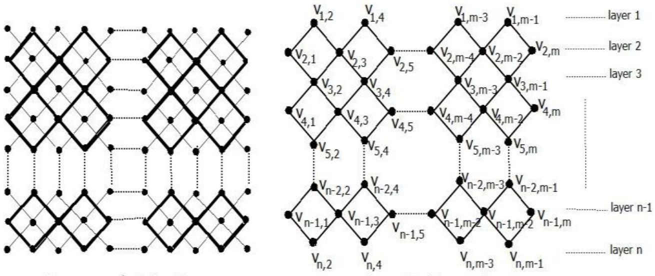


Figure 3.

II. EQUITABLE COLORING OF SUDHA GRID GRAPHS

Thoerem 2.1. *Sudha grid of diamonds $S_d(m,n)$ admit equitable coloring and its chromatic number is either 2 or 3 according to $|m-n| = 0$ (or 2) or $|m-n| > 2$.*

Proof. Sudha grid of diamonds $S_d(m,n)$ is the induced subgraph of the tensor product of the path P_m and the path P_n (for odd $m \geq 3$ and odd $n \geq 3$). The vertices of $S_d(m,n)$ are denoted by $v_{i,j}$, $1 \leq i \leq n, 1 \leq j \leq m$ as shown in figure 4.



Tensor product $P_m \times P_n$ Sudha gird of diamonds

Figure 4

There are two cases :

Case (i) : Let $|m-n| = 0$ or 2.

The function f from the vertex set of $S_d(m,n)$ to the set of colors $\{1,2\}$ is defined as

$$f(v_{i,j}) = \begin{cases} 1, & i \equiv 1 \pmod 2 \\ 2, & i \equiv 0 \pmod 2 \end{cases}$$

The vertices in odd layers are colored with 1 and the even layers are colored with 2 satisfying the condition $||C[1]| - |C[2]|| \leq 1$, since

(i) $|C[1]| = |C[2]| = \frac{mn-1}{4}$ when $m = n$,

(ii) $|C[1]| = \frac{mn+1}{4}$ and $|C[2]| = \frac{mn-3}{4}$ when $m = n-2$,

(iii) $|C[1]| = \frac{mn-3}{4}$ and $|C[2]| = \frac{mn+1}{4}$ when $m = n+2$.

Sudha grid of diamonds $S_d(m,n)$ has equitable coloring with this type of coloring and hence $\chi_=(S_d(m,n)) = 2$ if $|m-n| = 0$ or 2.

Case (ii) : Let $|m-n| > 2$.

There are three types depending on the value of m.

The function f from the vertex set of $S_d(m,n)$ to the set of colors $\{1,2,3\}$ is defined as follows :

Type (a) : Let $m = 1+6j$, $j = 1,2,3,\ldots$

The vertices of $S_d(m,n)$ are colored as for i odd, j even,

$$f(v_{i,j}) = \begin{cases} 3, & i \equiv 2 \pmod 6 \\ 1, & i \equiv 4 \pmod 6 \\ 2, & i \equiv 0 \pmod 6 \end{cases}$$

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for i even, j odd and $1 < j < m$,

$$f(v_{i,j}) = \begin{cases} 1, & i \equiv 1 \ (mod\ 6) \\ 2, & i \equiv 3 \ (mod\ 6) \\ 3, & i \equiv 5 \ (mod\ 6) \end{cases},$$

$$f(v_{i,1}) = \begin{cases} 1, & i \equiv 2, 0 \ (mod\ 6) \\ 2, & i \equiv 4 \ (mod\ 6) \end{cases}$$

and $f(v_{i,m}) = \begin{cases} 1, & i \equiv 2, 4 \ (mod\ 6) \\ 2, & i \equiv 0 \ (mod\ 6) \end{cases}$

The color classes $C[1], C[2]$ and $C[3]$ satisfy the conditions $||C[i]| - |C[j]|| \leq 1,\ 1 \leq i \leq 3, 1 \leq j \leq 3,$ since

(i) $|C[1]| = |C[2]| = \frac{mn-1}{4}$ when $m = n$,

(ii) $|C[1]| = \frac{mn+1}{4}$ and $|C[2]| = \frac{mn-3}{4}$ when $m = n - 2$,

(iii) $|C[1]| = \frac{mn-3}{4}$ and $|C[2]| = \frac{mn+1}{4}$ when $m = n + 2$.

Sudha grid of diamonds $S_d(m, n)$ has equitable coloring with this type of coloring and hence $\chi_=(S_d(m, n)) = 2$ if $|m - n| = 0$ or 2.

Type (b) : Let $m = 3 + 6j,\ j = 1, 2, 3, \ldots$

The vertices of $S_d(m, n)$ are colored as

for i odd, j even,

$$f(v_{i,j}) = \begin{cases} 2, & i \equiv 2 \ (mod\ 6) \\ 1, & i \equiv 4 \ (mod\ 6) \\ 3, & i \equiv 0 \ (mod\ 6) \end{cases}$$

for i even, j odd,

$$f(v_{i,j}) = \begin{cases} 1, & i \equiv 1 \ (mod\ 6) \\ 3, & i \equiv 3 \ (mod\ 6) \\ 2, & i \equiv 5 \ (mod\ 6) \end{cases}$$

The color classes $C[1], C[2]$ and $C[3]$ satisfy the conditions $||C[i]| - |C[j]|| \leq 1,\ 1 \leq i \leq 3, 1 \leq j \leq 3,$ since $|C[1]| = |C[3]| = \frac{mn-3}{6}$ and $|C[2]| = \frac{mn-3}{6}$.

Sudha grid of diamonds $S_d(m, n)$ has equitable coloring with this type of coloring and hence $\chi_=(S_d(m, n)) = 3$ if $m = 3 + 6j,\ j = 1, 2, 3, \ldots$.

Type (c) : Let $m = 5 + 6j,\ j = 1, 2, 3, \ldots$

The vertices of $S_d(m, n)$ are colored as

for i odd, j even,

$$f(v_{i,j}) = \begin{cases} 2, & i \equiv 2 \ (mod\ 6) \\ 3, & i \equiv 4 \ (mod\ 6) \\ 1, & i \equiv 0 \ (mod\ 6) \end{cases}$$

for i even, j odd and $1 < j < m$,

$$f(v_{i,j}) = \begin{cases} 3, & i \equiv 1 \ (mod\ 6) \\ 1, & i \equiv 3 \ (mod\ 6) \\ 2, & i \equiv 5 \ (mod\ 6) \end{cases},$$

$$f(v_{i,1}) = \begin{cases} 3, & i \equiv 2, 0 \ (mod\ 6) \\ 1, & i \equiv 4 \ (mod\ 6) \end{cases},$$

and $f(v_{i,m}) = \begin{cases} 1, & i \equiv 2 \ (mod\ 6) \\ 2, & i \equiv 4, 0 \ (mod\ 6) \end{cases}$.

The color classes $C[1], C[2]$ and $C[3]$ satisfy the conditions $||C[i]| - |C[j]|| \leq 1,\ 1 \leq i \leq 3, 1 \leq j \leq 3$, since

(i) $|C[1]| = |C[2]| = \frac{mn-3}{6}$ and $|C[3]| = \frac{mn+3}{6}$ when $n \equiv 0 \ (mod\ 3)$,

(ii) $|C[1]| = |C[2]| = |C[3]| = \frac{mn-1}{6}$ when $n \equiv 2 \ (mod\ 3)$,

(iii) $|C[1]| = \frac{mn-5}{6}$ and $|C[2]| = |C[3]| = \frac{mn+1}{6}$ when $n \equiv 1 \ (mod\ 3)$.

Sudha grid of diamonds $S_d(m, n)$ has equitable coloring with this type of coloring and hence $\chi_=(S_d(m, n)) = 3$ if $m = 5 + 6j,\ j = 1, 2, 3, \ldots$.

Therefore the equitable chromatic number of $S_d(m, n)$ is either 2 or 3 according to $|m - n| = 0\ (or\ 2)$ or $|m - n| > 2$.

Illustration 2.2.

Consider the graph $S_d(7, 5)$. Using theorem 2.1 case (i) we assign the color 1 to the vertices $v_{1,2}, v_{1,4}, v_{1,6}, v_{3,2}, v_{3,4}, v_{3,6}, v_{5,2}, v_{5,4}, v_{5,6}$ and color 2 to the vertices $v_{2,1}, v_{2,3}, v_{2,5}, v_{2,7}, v_{4,1}, v_{4,3}, v_{4,5}, v_{4,7}$ as shown in figure 5.

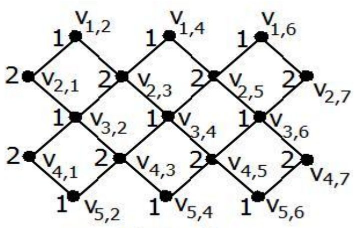

Figure 5

Here $|C[1]| = 9, |C[2]| = 8$ and satisfy the condition $||C[1]| - |C[2]|| < 1$. This type of coloring on Sudha gird of diamonds $S_d(7, 5)$ satisfy the conditions for equitable coloring. Hence $\chi_=(S_d(7, 5)) = 2$.

Illustration 2.3

Consider the graph $S_d(13, 7)$. Using theorem 2.1 case (ii) type (a) we assign the color 1 to the vertices $v_{1,4}, v_{1,10}, v_{2,1}, v_{2,7}, v_{2,13}, v_{3,4}, v_{3,10}, v_{4,7}, v_{4,13}, v_{5,4}, v_{5,10}, v_{6,1}, v_{6,7}, v_{7,4}, v_{7,10},$ color 2 to the vertices $v_{1,6}, v_{1,12}, v_{2,3}, v_{2,9}, v_{3,6}, v_{3,12}, v_{4,1} v_{4,3}, v_{4,9}, v_{5,6}, v_{5,12}, v_{6,3}, v_{6,9}, v_{7,6}, v_{7,12}$ and color 3 to the vertices $v_{1,1}, v_{1,8}, v_{2,5}, v_{2,11}, v_{3,2}, v_{3,8}, v_{4,5} v_{4,11}, v_{5,2}, v_{5,8}, v_{6,5}, v_{6,11}, v_{6,13}, v_{7,2}, v_{7,8}$ as shown in figure 6.

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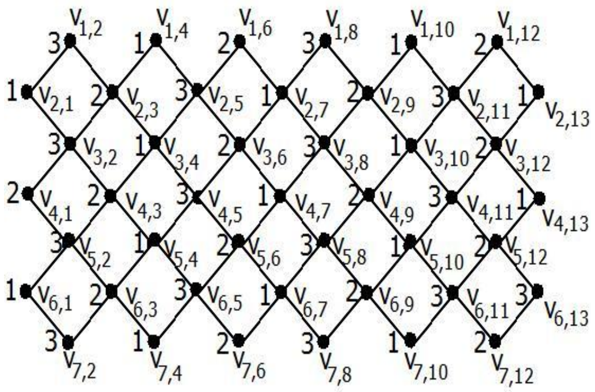

Figure 6

Here $|C[1]| = 15, |C[2]| = 15$ and $|C[3]| = 15$, and they satisfy the condition $||C[i]| - |C[j]|| < 1, 1 \leq i \leq 3, 1 \leq j \leq 3$. This type of coloring on Sudha gird of diamond $S_d(13,7)$ satisfy the conditions for equitable coloring. Hence $\chi_=(S_d(13,7)) = 3$.

Theorem 2.4. Sudha grid of hexagons $S_h(m,n)$ admit equitable coloring and its chromatic number is 2.

Proof. Sudha grid of hexagons $S_h(m,n)$ is the induced subgraph of the strong product of the path P_m and the path P_n (for odd $m \geq 3$ and $n \equiv 0 \ (mod\ 4)$). The vertices of $S_d(m,n)$ are denoted by $v_{i,j}$, $1 \leq i \leq n, 1 \leq j \leq m$ as shown in figure 7.

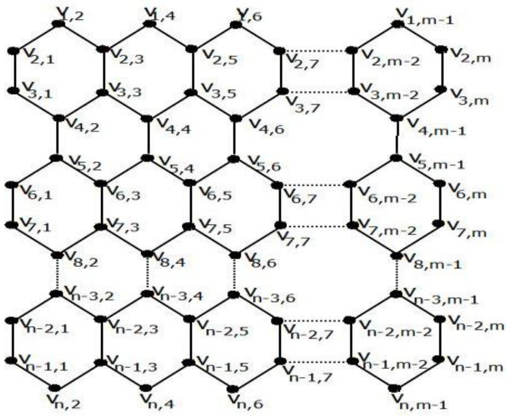

Figure 7

The function f from the vertex set of $S_h(m,n)$ to the set of colors $\{1,2\}$ is defined as

$$f(v_{i,j}) = \begin{cases} 1, & i \equiv 1 \ (mod\ 2) \\ 2, & i \equiv 0 \ (mod\ 2) \end{cases}$$

The color classes $C[1]$ and $C[2]$ satisfy the condition $||C[1]| - |C[2]|| \leq 1$. Sudha grid of hexagons $S_d(m,n)$ has equitable coloring with this type of coloring and hence $\chi_=(S_h(m,n)) = 2$.

Illustration 2.5

Consider the graph $S_h(7,8)$. Using theorem 2.4, we assign the color 1 to the vertices $v_{1,2}, v_{1,4}, v_{1,6}, v_{3,1}, v_{3,3}, v_{3,5}, v_{3,7}, v_{5,2}, v_{5,4}, v_{5,6}, v_{7,1}, v_{7,3}, v_{7,5}, v_{7,7}$ and color 2 to the vertices $v_{2,1}, v_{2,3}, v_{2,5}, v_{2,7}, v_{4,2}, v_{4,4}, v_{4,6}, v_{6,1}, v_{6,3}, v_{6,5}, v_{6,7}, v_{8,2}, v_{8,4}, v_{8,6}$ as shown in figure 8.

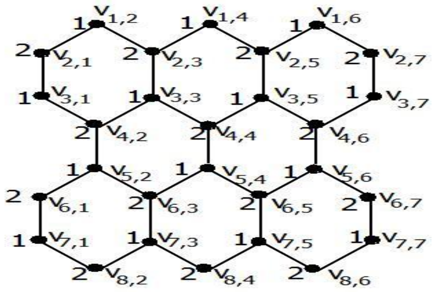

Figure 8

Here $|C[1]| = 14, |C[2]| = 14$ and they satisfy the condition $||C[1]| - |C[2]|| < 1$. This type of coloring on Sudha gird of hexagon $S_h(7,8)$ satisfy the condition for equitable coloring. Hence $\chi_=(S_d(7,8)) = 2$.

III. EQUITABLE COLORING OF SUDHA GRAPH

Theorem 3.1. Sudha graph $S(n,2)$ admits equitable coloring and its chromatic number is either 3 or 4 according to $n \equiv 0 \ (mod\ 3)$ or $n \not\equiv 0 \ (mod\ 3)$.

Proof. Let $v_1, v_2, v_3, \dots, v_{n-1}, v_n$ be the vertices of the graph $S(n,2)$ and its edges are defined as follows:

for $1 \leq i \leq n$,

(i) v_i is adjacent to v_{i+1} and v_n is adjacent to v_1

(ii) v_i is adjacent to v_{i+2} if $i + 2 < n$

(iii) v_i is adjacent to v_{i+n-2} if $i + 2 \geq n$ as shown in figure 9.

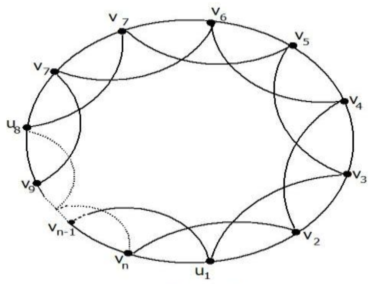

Figure 9

There are three cases :

The function f from the vertex set of $S(n,2)$ to the set of colors $\{1,2,3,4\}$ is defined as follows :

Case (i) : Let $n \equiv 0 \ (mod\ 3)$

The vertices of $S(n,2)$ are colored as

$$f(v_i) = \begin{cases} 1, & i \equiv 1 \ (mod\ 3) \\ 2, & i \equiv 2 \ (mod\ 3) \\ 3, & i \equiv 0 \ (mod\ 3) \end{cases}$$

for $1 \leq i \leq n$

The color classes $C[1]$, $C[2]$ and $C[3]$ satisfy the condition $||C[i]| - |C[j]|| \leq 1, 1 \leq i \leq 3, 1 \leq j \leq 3$, since $|C[1]| = |C[2]| = |C[3]| = \frac{n}{3}$.

Sudha graph $S(n,2)$ has equitable coloring with this type of coloring and hence $\chi_=(S(n,2)) = 3$ if $n \equiv 0\ (mod\ 3)$.

Case (ii) : Let n be odd and $n \not\equiv 0\ (mod\ 3)$

There are two types :

Type (a) : Let $n = 7 + 4j,\ j = 0, 1\ (mod\ 3)$

The vertices of $S(n,2)$ are colored as

$$f(v_i) = \begin{cases} 1, & i \equiv 1\ (mod\ 4) \\ 2, & i \equiv 2\ (mod\ 4) \\ 3, & i \equiv 3\ (mod\ 4) \\ 4, & i \equiv 0\ (mod\ 4) \end{cases}$$

for $1 \leq i \leq n$.

The color classes $C[1]$, $C[2]$ and $C[3]$ satisfy the condition $||C[i]| - |C[j]|| \leq 1, 1 \leq i \leq 4, 1 \leq j \leq 4$, since $|C[1]| = |C[2]| = |C[3]| = \frac{n+1}{4}$ and $|C[4]| = \frac{n-3}{4}$

Sudha graph $S(n,2)$ has equitable coloring with this type of coloring and hence $\chi_=(S(n,2)) = 4$ if $n = 7 + 4j,\ j = 0, 1\ (mod\ 3)$.

Type (b) : Let $n = 9 + 4j,\ j = 1, 2\ (mod\ 3)$

The vertices of $S(n,2)$ are colored as

$$f(v_i) = \begin{cases} 1, & i \equiv 1\ (mod\ 4) \\ 2, & i \equiv 2\ (mod\ 4) \\ 3, & i \equiv 3\ (mod\ 4) \\ 4, & i \equiv 0\ (mod\ 4) \end{cases}$$

for $1 \leq i < n - 4$,

$f(v_{n-4}) = 2$,
$f(v_{n-3}) = 3$,
$f(v_{n-2}) = 1$,
$f(v_{n-1}) = 2$
and $f(v_n) = 4$.

The color classes $C[1]$, $C[2]$ and $C[3]$ satisfy the condition $||C[i]| - |C[j]|| \leq 1,\ 1 \leq i \leq 4, 1 \leq j \leq 4$, since $|C[1]| = |C[3]| = |C[4]| = \frac{n-1}{4}$ and $|C[2]| = \frac{n+3}{4}$.

Sudha graph $S(n,2)$ has equitable coloring with this type of coloring and hence $\chi_=(S(n,2)) = 4$ if $n = 9 + 4j,\ j = 1, 2\ (mod\ 3)$.

Case (iii) : Let n be even and $n \not\equiv 0\ (mod\ 3)$

There are two types :

Type (a) : Let $n = 6 + 4j,\ j = 1, 2\ (mod\ 3)$

The vertices of $S(n,2)$ are colored as

$$f(v_i) = \begin{cases} 1, & i \equiv 1\ (mod\ 4) \\ 2, & i \equiv 2\ (mod\ 4) \\ 3, & i \equiv 3\ (mod\ 4) \\ 4, & i \equiv 0\ (mod\ 4) \end{cases}$$

for $1 \leq i < n - 1$,

$f(v_{n-1}) = 2$
and $f(v_n) = 3$.

The color classes $C[1]$, $C[2]$ and $C[3]$ satisfy the condition $||C[i]| - |C[j]|| \leq 1,\ 1 \leq i \leq 4, 1 \leq j \leq 4$, since $|C[1]| = |C[2]| = |C[3]| = |C[4]| = \frac{n}{2}$.

Sudha graph $S(n,m)$ has equitable coloring with this type of coloring and hence $\chi_=(S(n,2)) = 4$ if $n = 6 + 4j,\ j = 1, 2\ (mod\ 3)$.

Type (b) : Let $n = 8 + 4j,\ j = 0, 2\ (mod\ 3)$

The vertices of $S(n,2)$ are colored as

$$f(v_i) = \begin{cases} 1, & i \equiv 1\ (mod\ 4) \\ 2, & i \equiv 2\ (mod\ 4) \\ 3, & i \equiv 3\ (mod\ 4) \\ 4, & i \equiv 0\ (mod\ 4) \end{cases}$$

for $1 \leq i \leq n$.

The color classes $C[1]$, $C[2]$ and $C[3]$ satisfy the condition $||C[i]| - |C[j]|| \leq 1,\ 1 \leq i \leq 4, 1 \leq j \leq 4$, since $|C[1]| = |C[4]| = \frac{n-2}{4}$ and $|C[2]| = |C[3]| = \frac{n+2}{4}$.

Sudha graph $S(n,2)$ has equitable coloring with this type of coloring and hence $\chi_=(S(n,2)) = 4$ if $n = 8 + 4j,\ j = 0, 2\ (mod\ 3)$.

Therefore the equitable chromatic number of $S(n,2)$ is either 3 or 4 according to $n \equiv 0\ (mod\ 3)$ or $n \not\equiv 0\ (mod\ 3)$.

Illustration 3.2.

Consider the graph $S(9,2)$. Using theorem 3.1 case (i) we assign the color 1 to the vertices v_1, v_4, v_7, color 2 to the vertices v_2, v_5, v_8 and color 3 to the vertices v_3, v_6, v_9 as shown in figure 10.

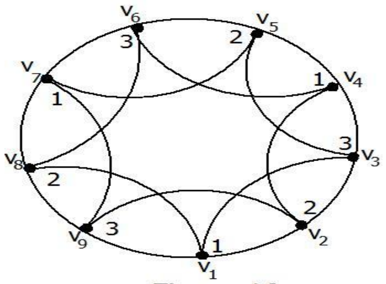

Figure 10

Here $|C[1]| = 3$, $|C[2]| = 3$ and $|C[3]| = 3$. They satisfy the conditions $||C[i]| - |C[j]|| < 1, 1 \leq i \leq 3$, $1 \leq j \leq 3$. This type of coloring on Sudha graph $S(9,2)$

satisfy the conditions for equitable coloring. Hence $\chi_{=}(S(9,2)) = 3$.

Illustration 3.3.

Consider the graph $S(17,2)$. Using theorem 3.1 case (ii) type (b), we assign the color 1 to the vertices v_1, v_5, v_9, v_{15} color 2 to the vertices $v_2, v_6, v_{10}, v_{13}, v_{16}$, color 3 to the vertices v_3, v_7, v_{11}, v_{14} and color 4 for the vertices v_4, v_8, v_{12}, v_{17} as shown in figure 11.

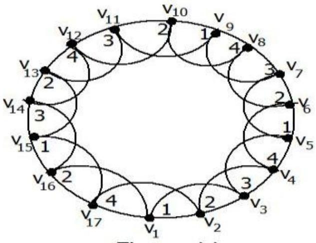

Figure 11

Here $|C[1]| = 4$, $|C[2]| = 5$, $|C[3]| = 4$ and $|C[4]| = 4$. They satisfy the condition $||C[i]| - |C[j]|| < 1, 1 \leq i \leq 4$, $1 \leq j \leq 4$. This type of coloring on Sudha graph $S(17,2)$ satisfy the conditions for equitable coloring. Hence $\chi_{=}(S(17,2)) = 4$.

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