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IVF-generalized semi-precontinuous mappings

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***Abstract*—In this paper, we introduce interval valued fuzzy(for- ivf) generalized semi-precontinuous mappings. Also we investigate some of their properties.**

Keywords*—: *IVF-set; IVFT-space; IVF-point; IVF-generalized semi-preclosed set.

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I. INTRODUCTION

The concept of fuzzy subset was introduced and studied by L. A. Zadeh [13] in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. The following papers have motivated us to work on this paper: C. L. Chang [3] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces. Many researchers like this concept and many others have contributed to the development of fuzzy topological spaces. Andrijevic [1] introduced semi-preclosed sets and Dontchev [4] introduced generalized semi-preclosed sets in general topology. After that the set was generalized to fuzzy topological spaces by Saraf and Khanna [12]. Tapas Kumar Mondal and S. K. Samantha [9] introduced the topology of interval valued fuzzy sets. Jeyabalan. R, Arjunan. K, [6] introduced interval valued fuzzy generalaized semi-preclosed sets . In this paper, we introduce that ivf-generalized semi-precontinuous mappings and some properties are investigated.

II. PRELIMINARIES

Definition 2.1. [9] *Let* X *be a non empty set. A mapping* $\overline{A}: X \to D[0,1]$ *is called an interval valued fuzzy set (briefly* $IVFS$ *) on* X *, where* $D[0,1]$ *denotes the family of all closed subintervals of [0,1] and* $\overline{A}(x) = [A^-(x), A^+(x)]$*, for all* $x \in X$ *, where* $A^-(x)$ *and* $A^+(x)$ *are fuzzy sets of* X *such that* $A^-(x) \le A^+(x)$*, for all* $x \in X$ *.*

Thus $\overline{A}(x)$ is an interval (a closed subset of [0,1]) and not a number fom the interval [0,1] as in the case of fuzzy set.Obviously any fuzzy set A on X is an $IVFS$.

Notation 2.2. D^X denotes the set of all IVF-subsets of a non empty set X .

Definition 2.3. [9] *Let* X *be a non empty set. Let* $x_0 \in X$ *and* $\alpha \in D[0,1]$ *be fixed such that* $\alpha \ne [0,0]$*. Then the IVF-subset* $p_{x_0}^{\alpha}$ *is called an IVF-point defined by,*

$$p_{x_0}^{\alpha} = \{\alpha \; x{=}x\_0 \text{ if } x \, x\_0 \}.$$

Definition 2.4. [9] *Let* $\overline{A}$ *and* $\overline{B}$ *be any two* $IVFS$ *of* X *, that is* $\overline{A} = \{< x, [A^-(x), A^+(x)] >: x \in X\}$, $\overline{B} = \{< x, [B^-(x), B^+(x)] >: x \in X\}$.
We define thefollowing relations and operations:

(*i*) $\overline{A} \subseteq \overline{B}$ *if and only if* $A^-(x) \le B^-(x)$ *and* $A^+(x) \le B^+(x)$*, for all* $x \in X$ *.*

(*ii*) $\overline{A} = \overline{B}$ *if and only if* $A^-(x) = B^-(x)$*, and* $A^+(x) = B^+(x)$*, for all* $x \in X$ *.*

(*iii*) $(\overline{A})^c = \overline{1} - \overline{A} = \{< x, [1 - A^+(x), 1 - A^-(x)] >: x \in X\}$.

(*iv*) $\overline{A} \cap \overline{B}$ $= \{< x, [\min[A^-(x), B^-(x)], \min[A^+(x), B^+(x)]] >: x \in X\}$.

(*v*) $\overline{A} \cup \overline{B}$ $= \{< x, [\max[A^-(x), B^-(x)], \max[A^+(x), B^+(x)]] >: x \in X\}$.

We denote by $\overline{0}_X$ and $\overline{1}_X$ for the IVF-sets $\{< x, [0,0] >, \textit{ for all } x \in X\}$ and $\{< x, [1,1] >, \textit{ for all } x \in X\}$ respectively.

Definition 2.5. [9] *Let* X *be a set and* $\Im$ *be a family of interval vlued fuzzy sets* $(IVFSs)$ *of* X *. The family* $\Im$ *is called an interval valued fuzzy topology* $(IVFT)$ *on* X *if and only if* $\Im$ *satisfies the following axioms:*

(*i*) $\overline{0}_X, \overline{1}_X \in \Im$,

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(ii) *If* $\{\overline{A}_i : i \in I\} \subseteq \Im$, *then* $\bigcup_{i\in I} \overline{A}_i \in \Im$,

(iii) *If* $\overline{A}_1, \overline{A}_2, \overline{A}_3, \ldots, \overline{A}_n \in \Im$, *then* $\bigcap_{i=1}^{n} \overline{A}_i \in \Im$.

The pair $(X, \Im)$ is called an IVFT-space $(IVFTS)$. The members of $\Im$ are called interval valued fuzzy open sets $(IVFOS)$ in X.

An interval valued fuzzy set $\overline{A}$ in X is said to be interval valued fuzzy closed set $(IVFCS)$ in X if and only if $(\overline{A})^c$ is an $IVFOS$ in X.

Definition 2.6. [9] *Let* $(X, \Im)$ *be an* $IVFTS$ *and* $\overline{A} = \{< x, [A^-(x), A^+(x)] >: x \in X\}$ *be an* $IVFS$ *in* X. *Then the interval valued fuzzy interior and interval valued fuzzy closure of* $\overline{A}$ *denoted by* $IVFint(\overline{A})$ *and* $IVFcl(\overline{A})$ *are defined by*

$$IVFint(\overline{A}) = \bigcup\{\overline{G} : \overline{G} \text{ is an IVFOS in } X \text{ and } \overline{G} \subseteq \overline{A}\}$$

$$IVFcl(\overline{A}) = \bigcap\{\overline{K} : \overline{K} \text{ is an IVFCS in } X \text{ and } \overline{A} \subseteq \overline{K}\}.$$

For any $IVFS$ $\overline{A}$ in $(X, \Im)$, we have $IVFcl(\overline{A}^c) = (IVFint\ (\overline{A}))^c$ and $IVFint(\overline{A}^c) = (IVFcl(\overline{A}))^c$.

Definition 2.7.

An $IVFS$ $\overline{A} = \{< x, [A^-(x), A^+(x)] >: x \in X\}$ *in an* $IVFTS$ $(X, \Im)$ *is said to be an*

(i) *IVF-regular closed set* ($IVFRCS$) *if* $\overline{A} = IVFcl(IVFint(\overline{A}))$;

(ii) *IVF-semi closed set* ($IVFSCS$) *if* $IVFint(IVFcl(\overline{A})) \subseteq \overline{A}$;

(iii) *IVF-preclosed set* ($IVFPCS$) *if* $IVFcl(IVFint\ (\overline{A})) \subseteq \overline{A}$;

(iv) *IVF-*α *closed set* ($IVF\alpha CS$) *if* $IVFcl(IVFint\ (IVFcl\ (\overline{A}))) \subseteq \overline{A}$;

(v) *IVF-*β *closed set* ($IVF\beta CS$) *if* $IVFint(ivfcl\ (ivfint(\overline{A}))) \subseteq \overline{A}$.

Definition 2.8.

An $IVFS$ $\overline{A} = \{< x, [A^-(x), A^+(x)] >: x \in X\}$ *in an* $IVFTS$ $(X, \Im)$ *is said to be an*

(i) *interval valued fuzzy generalized closed set* ($IVFGCS$) *if* $ivfcl\ (\overline{A}) \subseteq \overline{U}$, *whenever* $\overline{A} \subseteq \overline{U}$ *and* $\overline{U}$ *in an* $IVFOS$;

(ii) *interval valued fuzzy regular generalized closed set* ($IVFRGCS$) *if* $ivfcl\ (\overline{A}) \subseteq \overline{U}$, *whenever* $\overline{A} \subseteq \overline{U}$ *and* $\overline{U}$ *is an* $IVFROS$.

Definition 2.9.

An $IVFS$ $\overline{A} = \{< x, [A^-(x), A^+(x)] >: x \in X\}$ *in an* $IVFTS$ $(X, \Im)$ *is said to be an*

(i) *interval valued fuzzy semi-preclosed set* ($IVFSPCS$) *if there exist an* $IVFPCS$ $\overline{B}$, *such that* $ivfint\overline{B} \subseteq \overline{A} \subseteq \overline{B}$;

(ii) *interval valued fuzzy semi-preopen set* ($IVFSPOS$) *if there exist an* $IVFPOS$ $\overline{B}$, *such that* $\overline{B} \subseteq \overline{A} \subseteq ivfcl(\overline{B})$.

Definition 2.10. *Let* $\overline{A}$ *be an* $IVFS$ *in an* $IVFTS$ $(X, \Im)$. *Then the interval valued fuzzy semi-preinterior of* $\overline{A}$ $(ivfspint(\overline{A}))$ *and the interval valued fuzzy semi-preclosure of* $\overline{A}(ivfspcl(\overline{A}))$ *are defined by*

$$ivfspint(\overline{A}) = \bigcup\{\overline{G} : \overline{G} \text{ is an IVFSPOS in } X \text{ and } \overline{G} \subseteq \overline{A}\},$$

$$ivfspcl(\overline{A}) = \bigcap\{\overline{K} : \overline{K} \text{ is an IVFSPCS in } X \text{ and } \overline{A} \subseteq \overline{K}\}.$$

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For any $IVFS$ $\overline{A}$ in $(X, \Im)$, we have $ivfspcl(\overline{A}^c) = (ivfspint(\overline{A}))^c$ and $ivfspint(\overline{A}^c) = (ivfspcl(\overline{A}))^c$.

Definition 2.11. [6] *An* $IVFS$ $\overline{A}$ *in* $IVFTS$ $(X, \Im)$ *is said to be an interval valued fuzzy generalized semi-preclosed set* ($IVFGSPCS$) *if* $ivfspcl\ (\overline{A}) \subseteq \overline{U}$, *whenever* $\overline{A} \subseteq \overline{U}$ *and* $\overline{U} \in \Im$.

Definition 2.12. [6] *The complement* $\overline{A}^c$ *of an* $IVFGSPCS$ $\overline{A}$ *in an* $IVFTS$ $(X, \Im)$ *is called an interval valued fuzzy generalized semi-preopen set* ($IVFGSPOS$) *in* X.

Definition 2.13. *An* $IVFTS$ $(X, \Im)$ *is called an interval valued fuzzy* $T_{1/2}$ *space* $(IVFT_{1/2})$ *space if every* $IVFGCS$ *is an* $IVFCS$ *in* X.

Definition 2.14. *An* $IVFTS$ $(X, \Im)$ *is called an interval valued fuzzy semi-pre* $T_{1/2}$ *space* $(IVFSPT_{1/2})$ *space if every* $IVFGSPCS$ *is an* $IVFSPCS$ *in* X.

Definition 2.15. [9] *An* $IVFS$ $\overline{A}$ *of a* $IVFTS$ *of* $(X, \Im)$ *is said to be an interval valued fuzzy neighbourhood(* $IVFN$ *) of an* $IVFP$ $p_{x_0}^{\alpha}$ *if there exists an* $IVFOS$ $\overline{B}$ *in* X *such that* $p_{x_0}^{\alpha} \in \overline{B} \subseteq \overline{A}$.

Definition 2.16. *Let* $(X,\Im)$ *and* (Y,σ) *be* $IVFTSs$. *Then a map* $g: X \to Y$ *is called an*

(i) *interval valued fuzzy continuous* (IVF *continuous mapping) if* $g^{-1}(\overline{B})$ *is* $IVFOS$ *in* X *for all* $\overline{B}$ *in* σ.

(ii) *interval valued fuzzy semi-continuous mapping* ($IVFS$ *-continuous mapping) if* $g^{-1}(\overline{B})$ *is* $IVFSOS$ *in* X *for all* $\overline{B}$ *in* σ.

(iii) *interval valued fuzzy* α*-continuous mapping* ($IVF\alpha$*-continuous mapping) if* $g^{-1}(\overline{B})$ *is* $IVF\alpha OS$ *in* X *for all* $\overline{B}$ *in* σ.

(iv) *interval valued fuzzy pre-continuous mapping* ($IVFP$*-continuous mapping) if* $g^{-1}(\overline{B})$ *is* $IVFPOS$ *in* X *for all* $\overline{B}$ *in* σ.

(v) *interval valued fuzzy* β*-continuous mapping* ($IVF\beta$*-continuous mapping) if* $g^{-1}(\overline{B})$ *is* $IVF\beta OS$ *in* X *for all* $\overline{B}$ *in* σ.

Definition 2.17. *Let* $(X,\Im)$ *and* (Y,σ) *be* $IVFTSs$. *Then a map* $g: X \to Y$ *is called interval valued fuzzy generalized continuous* ($IVFG$ *continuous) mapping if* $g^{-1}(\overline{B})$ *is* $IVFGCS$ *in* X *for all* $\overline{B}$ *in* σ^c.

Definition 2.18. *A mapping* $g:(X,\Im)\to(Y,\sigma)$ *is called an interval valued fuzzy generalized semi-precontinuous* ($IVFGSP$ *continuous) mapping if* $g^{-1}(\overline{V})$ *is an* $IVFGSPCS$ *in* X *for every* $IVFCS$ $\overline{V}$ *in* Y.

Example 2.19. Let $X=\{a,b\}$, $Y=\{u,v\}$ and

$$\overline{K}_1=\{<a,[0.1,0.2]>,<b,[0.3,0.4]>\},$$
$$\overline{L}_1=\{<u,[0.3,0.4]>,<v,[0.4,0.6]>\}.$$

Then $\Im=\{\overline{0}_X,\overline{K}_1,\overline{1}_X\}$ and $\sigma=\{\overline{0}_Y,\overline{L}_1,\overline{1}_Y\}$ are $IVFT$ on X and Y respectively. Define a mapping $g:(X,\Im)\to(Y,\sigma)$ by $g(a)=u$ and $g(b)=v$. Then g is an $IVFGSP$ continuous mapping.

III. Main Results

Theorem 3.1. *Every* IVF *continuous mapping is an* $IVFGSP$ *continuous mapping.*

Proof. Let $g:(X,\Im)\to(Y,\sigma)$ be an IVF continuous mapping. Let $\overline{V}$ be an $IVFCS$ in Y. Then $g^{-1}(\overline{V})$ is an $IVFCS$ in X. Since every $IVFCS$ is an $IVFGSPCS$, $g^{-1}(\overline{V})$ is an $IVFGSPCS$ in X. Hence g is an $IVFGSP$ continuous mapping.

Remark 3.2. The converse of the above theorem 3.1. need not be true from the following example: Let $X=\{a,b\}$, $Y=\{u,v\}$ and

$$\overline{K}_1=\{<a,[0.1,0.2]>,<b,[0.3,0.4]>\},$$
$$\overline{L}_1=\{<u,[0.8,0.9]>,<v,[0.6,0.7]>\}.$$

Then $\Im=\{\overline{0}_X,\overline{K}_1,\overline{1}_X\}$ and $\sigma=\{\overline{0}_Y,\overline{L}_1,\overline{1}_Y\}$ are $IVFTs$ on X and Y respectively. Define a mapping $g:(X,\Im)\to(Y,\sigma)$ by $g(a)=u$ and $g(b)=v$. Then g is an $IVFGSP$ continuous mapping but not an IVF continuous mapping.

Theorem 3.3. *Every* $IVFG$ *continuous mapping is an* $IVFGSP$ *continuous mapping.*

Proof. Let $g:(X,\Im)\to(Y,\sigma)$ be an $IVFG$ continuous mapping. Let $\overline{V}$ be an $IVFCS$ in Y. Then $g^{-1}(\overline{V})$ is an $IVFGCS$ in X. Since every $IVFGCS$ is an $IVFGSPCS$, $g^{-1}(\overline{V})$ is an $IVFGSPCS$ in X. Hence g is an $IVFGSP$ continuous mapping.

Remark 3.4. The converse of the above theorem 3.3. need not be true from the following example: Let $X=\{a,b\}$, $Y=\{u,v\}$ and

$$\overline{K}_1=\{<a,[0.4,0.5]>,<b,[0.6,0.7]>\},$$
$$\overline{L}_1=\{<u,[0.6,0.7]>,<v,[0.7,0.8]>\}.$$

Then $\Im=\{\overline{0}_X,\overline{K}_1,\overline{1}_X\}$ and $\sigma=\{\overline{0}_Y,\overline{L}_1,\overline{1}_Y\}$ are $IVFTs$ on X and Y respectively. Define a mapping $g:(X,\Im)\to(Y,\sigma)$ by $g(a)=u$ and $g(b)=v$. Then g is an $IVFGSP$ continuous mapping but not an $IVFG$ continuous mapping. Since $\overline{L}_1^c$ $=\{<u,[0.3,0.4]>,<v,[0.2,0.3]>\}$ is an $IVFCS$ in Y and $g^{-1}(\overline{L}_1^c)=\{<a,[0.3,0.4]>,<b,[0.2,0.3]>\}\subseteq\overline{K}_1$. But $ivfcl(g^{-1}(\overline{L}_1^c))=\overline{K}_1^c\not\subset\overline{K}_1$. Therefore $g^{-1}(\overline{L}_1^c)$ is not an $IVFGCS$ in X.

Theorem 3.5. *Every* $IVFP$ *continuous mapping is an* $IVFGSP$ *continuous mapping.*

Proof. Let $g:(X,\Im)\to(Y,\sigma)$ be an $IVFP$ continuous mapping. Let $\overline{V}$ be an $IVFCS$ in Y. Then $g^{-1}(\overline{V})$ is an $IVFPCS$ in X. Since every $IVFPCS$ is an $IVFGSPCS$, $g^{-1}(\overline{V})$ is an $IVFGSPCS$ in X. Hence g is an $IVFGSP$ continuous mapping.

Remark 3.6. The converse of the above theorem 3.5 need not be true from the following example: Let $X=\{a,b\}$, $Y=\{u,v\}$ and

$$\overline{K}_1=\{<a,[0.3,0.4]>,<b,[0.4,0.7]>\},$$

$\overline{L}_1 = \{< u,[0.1,0.2] >, < v,[0.3,0.4] >\}$.

Then $\Im = \{\overline{0}_X, \overline{K}_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L}_1, \overline{1}_Y\}$ are *IVFTs* on X and Y respectively. Define a mapping $g:(X,\Im) \to (Y,\sigma)$ by $g(a) = u$ and $g(b) = v$. Then g is an *IVFGSP* continuous mapping but not an *IVFP* continuous mapping. Since $\overline{L}_1^c = \{< u,[0.8,0.9] >, < v,[0.6,0.7] >\}$ is an *IVFCS* in Y and $g^{-1}(\overline{L}_1^c) = \{< a,[0.8,0.9] >, < b,[0.6,0.7] >\}$ is not an *IVFPCS* in X, because $ivfcl(ivfint(g^{-1}(\overline{L}_1^c))) = ivfcl(\overline{K}_1) = \overline{1}_X \not\subset g^{-1}(\overline{L}_1^c)$.

Theorem 3.7. *Every* $IVF\beta$ *continuous mapping is an* *IVFGSP* *continuous mapping.*

Proof. Let $f:(X,\Im) \to (Y,\sigma)$ be an $IVF\beta$ continuous mapping. Let $\overline{V}$ be an *IVFCS* in Y. Then $g^{-1}(\overline{V})$ is an $IVF\beta CS$ in X. Since every $IVF\beta CS$ is an *IVFGSPCS*, $g^{-1}(\overline{V})$ is an *IVFGSPCS* in X. Hence g is an *IVFGSP* continuous mapping.

Remark 3.8. The converse of the above theorem 3.7. need not be true from the following example: Let $X = \{a,b\}$, $Y = \{u,v\}$ and

$\overline{K}_1 = \{< a,[0.5,0.7] >, < b,[0.3,0.4] >\}$,

$\overline{L}_1 = \{< u,[0.3,0.4] >, < v,[0.4,0.6] >\}$.

Then $\Im = \{\overline{0}_X, \overline{K}_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L}_1, \overline{1}_Y\}$ are *IVFTs* on X and Y respectively. Define a mapping $g:(X,\Im) \to (Y,\sigma)$ by $g(a) = u$ and $g(b) = v$. Then g is an *IVFGSP* continuous mapping but not an $IVF\beta$ continuous mapping. Since $\overline{L}_1^c = \{< u,[0.6,0.7] >, < v,[0.4,0.6] >\}$ is an *IVFCS* in Y and $g^{-1}(\overline{L}_1^c) = \{< a,[0.6,0.7] >, < b,[0.4,0.6] >\}$ is not an $IVF\beta CS$ in X, because $ivfint(ivfcl(ivfint(g^{-1}(\overline{L}_1^c)))) = ivfint(ivfcl(\overline{K}_1)))$ $= ivfint(\overline{1}_X) = \overline{1}_X \not\subset g^{-1}(\overline{L}_1^c)$

Theorem 3.9. *Every* $IVF\alpha$ *continuous mapping is an* *IVFGSP* *continuous mapping.*

Proof. Let $g:(X,\Im) \to (Y,\sigma)$ be an $IVF\alpha$ continuous mapping. Let $\overline{V}$ be an *IVFCS* in Y. Then $g^{-1}(\overline{V})$ is an $IVF\alpha CS$ in X. Since every $IVF\alpha CS$ is an *IVFGSPCS*, $g^{-1}(\overline{V})$ is an *IVFGSPCS* in X. Hence g is an *IVFGSP* continuous mapping.

Remark 3.10. The converse of the above theorem 3.9. need not be true from the following example: Let $X = \{a,b\}$, $Y = \{u,v\}$ and

$\overline{K}_1 = \{< a,[0.3,0.4] >, < b,[0.4,0.6] >\}$,

$\overline{L}_1 = \{< u,[0.1,0.2] >, < v,[0.3,0.4] >\}$.

Then $\Im = \{\overline{0}_X, \overline{K}_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L}_1, \overline{1}_Y\}$ are *IVFT* on X and Y respectively. Define a mapping $g:(X,\Im) \to (Y,\sigma)$ by $g(a) = u$ and $g(b) = v$. Then g is an *IVFGSP* continuous mapping but not an $IVF\alpha$ continuous mapping. Since $\overline{L}_1^c = \{< u,[0.8,0.9] >, < v,[0.6,0.7] >\}$ is an *IVFCS* in Y and $g^{-1}(\overline{L}_1^c) = \{< a,[0.8,0.9] >, < b,[0.6,0.7] >\}$ is not an $IVF\alpha CS$ in X, because $ivfcl(ivfint(ivfcl(g^{-1}(\overline{L}_1^c)))) = ivfcl(ivfint(\overline{1}_X))$. $= ivfcl(\overline{1}_X) = \overline{1}_X \not\subset g^{-1}(\overline{L}_1^c)$

Theorem 3.11. *Let* $g:(X,\Im) \to (Y,\sigma)$ *be a mapping where* $g^{-1}(\overline{V})$ *is an* *IVFRCS* *in* X, *for every* *IVFCS* $\overline{V}$ *in* Y. *Then* g *is an* *IVFGSP* *continuous mapping.*

Proof. Assume that $g:(X,\Im) \to (Y,\sigma)$ is a mapping. Let $\overline{A}$ be an *IVFCS* in Y. Then $g^{-1}(\overline{V})$ is an *IVFRCS* in X, by hypothesis. Since every *IVFRCS* is an *IVFGSPCS*, $g^{-1}(\overline{V})$ is an *IVFGSPCS* in X. Hence g is an *IVFGSP* continuous mapping.

Remark 3.12. The converse of the above theorem 3.11. need not be true from the following example: Let $X = \{a,b\}$, $Y = \{u,v\}$ and

$\overline{K}_1 = \{< a,[0.3,0.4] >, < b,[0.4,0.7] >\}$,

$\overline{L}_1 = \{< u,[0.1,0.2] >, < v,[0.3,0.4] >\}$.

Then $\Im = \{\overline{0}_X, \overline{K}_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L}_1, \overline{1}_Y\}$ are *IVFT* on X and Y respectively. Define a mapping $g:(X,\Im) \to (Y,\sigma)$ by $g(a) = u$ and $g(b) = v$. Then g is an *IVFGSP* continuous mapping but not a mapping as defined in theorem 3.11., since $\overline{L}_1^c = \{< u,[0.8,0.9] >, < v,[0.6,0.7] >\}$ is an *IVFCS* in Y and $g^{-1}(\overline{L}_1^c) = \{< a,[0.8,0.9] >, < b,[0.6,0.7] >\}$ is not an *IVFRCS* in X, because $ivfcl(ivfint(g^{-1}(\overline{L}_1^c))) = ivfcl(\overline{K}_1)) = \overline{1}_X \neq g^{-1}(\overline{L}_1^c)$.

Theorem 3.13. *Every* *IVFS* *continuous mapping is an* *IVFGSP* *continuous mapping.*

Proof. Let $g:(X,\Im) \to (Y,\sigma)$ be an *IVFG* continuous mapping. Let $\overline{V}$ be an *IVFCS* in Y. Then $g^{-1}(\overline{V})$ is an *IVFSCS* in X. Since every *IVFSCS* is an *IVFGSPCS*, $g^{-1}(\overline{V})$ is an *IVFGSPCS* in X. Hence g is an *IVFGSP* continuous mapping.

Theorem 3.14. *Every $IVFSP$ continuous mapping is an $IVFGSP$ continuous mapping.*

Proof. Let $g:(X,\Im)\to(Y,\sigma)$ be an $IVFSP$ continuous mapping. Let $\overline{V}$ be an $IVFCS$ in Y. Then $g^{-1}(\overline{V})$ is an $IVFSPCS$ in X. Since every $IVFSPCS$ is an $IVFGSPCS$, $g^{-1}(\overline{V})$ is an $IVFGSPCS$ in X. Hence g is an $IVFGSP$ continuous mapping.

Theorem 3.15. *Let $g:(X,\Im)\to(Y,\sigma)$ be an $IVFGSP$ continuous mapping, then for each $IVFP$ $p_{x_0}^{\alpha}$ of X and each $\overline{A}\in\sigma$ such that $g(p_{x_0}^{\alpha})\in\overline{A}$, there exist an $IVFGSPOS$ $\overline{B}$ of X such that $p_{x_0}^{\alpha}\in\overline{B}$ and $g(\overline{B})\subseteq\overline{A}$.*

Proof. Let $p_{x_0}^{\alpha}$ be an $IVFP$ of X and $\overline{A}\in\sigma$ such that $g(p_{x_0}^{\alpha})\in\overline{A}$. Put $\overline{B}=g^{-1}(\overline{A})$. Then by hypothesis, $\overline{B}$ is an $IVFGSPOS$ in X such that $p_{x_0}^{\alpha}\in\overline{B}$ and $g(\overline{B})=g(g^{-1}(\overline{A}))\subseteq\overline{A}$.

Theorem 3.16. *Let $g:(X,\Im)\to(Y,\sigma)$ be an $IVFGSP$ continuous mapping. Then g is an $IVFSP$ continuous mapping if X is an $IVFSPT_{1/2}$ space.*

Proof. Let $\overline{V}$ be an $IVFCS$ in Y. Then $g^{-1}(\overline{V})$ is an $IVFGSPCS$ in X, by hypothesis. Since X is an $IVFSPT_{1/2}$ space, $g^{-1}(\overline{V})$ is an $IVFSPCS$ in X. Hence g is an $IVFSP$ continuousmapping.

Theorem 3.17. *Let $g:(X,\Im)\to(Y,\sigma)$ be an $IVFGSP$ continuous mapping andlet $h:(Y,\sigma)\to(Z,\eta)$ be an $IVFG$ continuous mapping where Y is an $IVFT_{1/2}$ space. Then $h\circ g:(X,\Im)\to(Z,\eta)$ is an $IVFGSP$ continuous mapping.*

Proof. Let $\overline{V}$ be an $IVFCS$ in Z. Then $h^{-1}(\overline{V})$ is an $IVFGCS$ in Y, by hypothesis. Since Y is an $IVFT_{1/2}$ space, $h^{-1}(\overline{V})$ is an $IVFCS$ in Y. Therefore $g^{-1}(h^{-1}(\overline{V}))$ is an $IVFGSPCS$ in X, by hypothesis. Hence $h\circ g$ is an $IVFGSP$ continuous mapping.

Theorem 3.18. *For any $IVFS$ $\overline{A}$ in $(X,\Im)$ where X is an $IVFSPT_{1/2}$ space, $\overline{A}\in$ $IVFGSPO(X)$ if and only if for every $IVFP$ $p_{x_0}^{\alpha}\in\overline{A}$, there exists an $IVFGSPOS$ $\overline{B}$ in X such that $p_{x_0}^{\alpha}\in\overline{B}\subseteq\overline{A}$.*

Proof. *Necessity* : If $\overline{A}\in IVFGSPO(X)$, then we can take $\overline{B}=\overline{A}$ so that $p_{x_0}^{\alpha}\in\overline{B}\subseteq\overline{A}$ for every $IVFP$ $p_{x_0}^{\alpha}\in\overline{A}$.

Sufficiency : Let $\overline{A}$ be an $IVFS$ in $(X,\Im)$ and assume that there exist an $IVFGSPOS$ $\overline{B}$ in X such that $p_{x_0}^{\alpha}\in\overline{B}\subseteq\overline{A}$. Since X is an $IVFSPT_{1/2}$ space, $\overline{B}$ is an $IVFSPOS$ in X. Then $\overline{A}=\bigcup_{p_{x_0}^{\alpha}\in\overline{A}}\{p_{x_0}^{\alpha}\}\subseteq\bigcup_{p_{x_0}^{\alpha}\in\overline{A}}\overline{B}\subseteq\overline{A}$. Therefore $\overline{A}=\bigcup_{p_{x_0}^{\alpha}\in\overline{A}}\overline{B}$, which is an $IVFSPOS$ in X. Since $IVFSPOS$ is an $IVFGSPOS$. $\overline{A}$ is an $IVFGSPOS$ in $(X,\Im)$.

Theorem 3.19. *Let $g:(X,\Im)\to(Y,\sigma)$ be a mapping from $IVFT$ X into $IVFT$ Y. Then the following statements are equivalent if X and Y are $IVFSPT_{1/2}$ space:*

(i) g is an $IVFGSP$ continuous mapping,

(ii) $g^{-1}(\overline{B})$ is an $IVFGSPOS$ in X for each $IVFOS$ $\overline{B}$ in Y,

(iii) for every $IVFP$ $p_{x_0}^{\alpha}$ in X and for every $IVFOS$ $\overline{B}$ in Y such that $g(p_{x_0}^{\alpha})\in\overline{B}$, there exists an $IVFGSPOS$ $\overline{A}$ in X such that $p_{x_0}^{\alpha}\in\overline{A}$ and $g(\overline{A})\subseteq\overline{B}$.

Proof. $(i)\Leftrightarrow(ii)$ is obvious, since $g^{-1}(\overline{A}^c)=(g^{-1}(\overline{A}))^c$.

$(ii)\Rightarrow(iii)$ Let $\overline{B}$ be any $IVFOS$ in Y and let $p_{x_0}^{\alpha}\in D^X$. Given $g(p_{x_0}^{\alpha})\in\overline{B}$. By hypothesis $g^{-1}(\overline{B})$ is an $IVFGSPOS$ in X. Take $\overline{A}=g^{-1}(\overline{B})$. Now $p_{x_0}^{\alpha}\in g^{-1}(g(p_{x_0}^{\alpha}))$. Therefore $g^{-1}(g(p_{x_0}^{\alpha})\in g^{-1}(\overline{B}))=\overline{A}$. This implies $p_{x_0}^{\alpha}\in\overline{A}$ and $g(\overline{A})=g(g^{-1}(\overline{B}))\subseteq\overline{B}$.

$(iii)\Rightarrow(i)$ Let $\overline{A}$ be an $IVFCS$ in Y. Then its complement, say $\overline{B}=\overline{A}^c$ is an $IVFOS$ in Y. Let $p_{x_0}^{\alpha}\in D^X$ and $g(p_{x_0}^{\alpha})\in\overline{B}$. Then there exists an $IVFGSPOS$, say $\overline{C}$ in X such that $p_{x_0}^{\alpha}\in\overline{C}$ and $g(\overline{C})\subseteq\overline{B}$. Now $\overline{C}\subseteq g^{-1}(g(\overline{C}))\subseteq g^{-1}(\overline{B})$. Thus $p_{x_0}^{\alpha}\in g^{-1}(\overline{B})$. Therefore $g^{-1}(\overline{B})$ is an $IVFGSPOS$ in X, by theorem v. That is $g^{-1}(\overline{A}^c)$ is an $IVFGSPOS$ in

X and hence $g^{-1}(\overline{A})$ is an *IVFGSPCS* in X. Thus g is an *IVFGSP* continuous mapping.

Theorem 3.20. *Let $g:(X,\Im)\to(Y,\sigma)$ be a mapping from IVFT X into IVFT Y. Then the following statements are equivalent if X and Y are $IVFSPT_{1/2}$ space:*

(i) g is an IVFGSP continuous mapping,

(ii) for each IVFP $p_{x_0}^{\alpha}$ in X and for every IVFN $\overline{A}$ of $g(p_{x_0}^{\alpha})$, there exists an IVFGSPOS $\overline{B}$ in X such that $p_{x_0}^{\alpha}\in\overline{B}\subseteq g^{-1}(\overline{A})$,

(iii) for each IVFP $p_{x_0}^{\alpha}$ in X and for every IVFN $\overline{A}$ of $g(p_{x_0}^{\alpha})$, there exists an IVFGSPOS $\overline{B}$ in X such that $p_{x_0}^{\alpha}\in\overline{B}$ and $g(\overline{B})\subseteq\overline{A}$.

Proof. $(i)\Leftrightarrow(ii)$ Let $p_{x_0}^{\alpha}\in X$ and let $\overline{A}$ be an *IVFN* of $g(p_{x_0}^{\alpha})$. Then there exist an *IVFOS* $\overline{C}$ in Y such that $g(p_{x_0}^{\alpha})\in\overline{C}\subseteq\overline{A}$. Since g is an *IVFGSP* continuous mapping, $g^{-1}(\overline{C})=\overline{B}(say)$, is an *IVFGSPOS* in X and $p_{x_0}^{\alpha}\in\overline{B}\subseteq g^{-1}(\overline{A})$

$(ii)\Rightarrow(iii)$ Let $p_{x_0}^{\alpha}\in X$ and let $\overline{A}$ be an *IVFN* of $g(p_{x_0}^{\alpha})$. Then there exist an *IVFGSPOS* $\overline{B}$ in X such that $p_{x_0}^{\alpha}\in\overline{B}\subseteq g^{-1}(\overline{A})$, by hypothesis. Therefore $p_{x_0}^{\alpha}\in\overline{B}$ and $g(\overline{B})\subseteq g(g^{-1}(\overline{B}))\subseteq\overline{A}$.

$(iii)\Rightarrow(i)$ Let $\overline{B}$ be an *IVFOS* in Y and let $p_{x_0}^{\alpha}\in g^{-1}(\overline{B})$. Then $g(p_{x_0}^{\alpha})\in\overline{B}$. Therefore $\overline{B}$ is an *IVFN* of $g(p_{x_0}^{\alpha})$. Since $\overline{B}$ is an *IVFOS*, by hypothesis there exists an *IVFGSPOS* $\overline{A}$ in X such that $p_{x_0}^{\alpha}\in\overline{A}\subseteq g^{-1}(g(\overline{A}))\subseteq g^{-1}(\overline{B})$. Therefore $g^{-1}(\overline{B})$ is an *IVFGSPOS* in X, by theorem 3.18. Hence g is an *IVFGSP* continuous mapping.

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