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Properties of (α, β) cut sets of Intuitionistic L-Fuzzy Semi Filter (ILFSF) of lattices

*A. Maheswari**
Department of Mathematics,
Velammal College of Engineering & Technology,
Madurai, Tamil Nadu, India.
Email: sathyapriya1972@gmail.com

S. Karthikeyan
Assistant Professor/Mathematics
Velammal College of Engineering & Technology,
Madurai, Tamil Nadu, India.
Email: karthikeyan19732002@yahoo.com

M. Palanivelrajan
Department of Mathematics
Government Arts College,
Paramakudi, Tamil Nadu, India.
Email: palanivelrajan1975@gmail.com

***Abstract*— By combining the concept of L-fuzzy semi filter and Intuitionistic L-fuzzy sets we developed Intuitionistic L fuzzy Semi filter(ILFSF). As an extension of Intuitionistic L-fuzzy semi filter [3] the notion of (α, β) cut set have been put forward in our work. Some related properties also have been established with proof.**

Keywords*— Fuzzy subset, L-fuzzy subset, Intuitionistic L-fuzzy subset, Intuitionistic L-fuzzy semi filter, Intuitionistic L-fuzzy semi ideal, (α, β) cut sets of ILFSF

I. Introduction

In 1965, Lofti A. Zadeh [9] introduced the notion of a fuzzy subset of a set as a method for representing uncertainty in real physical world. The concept of intuitionistic fuzzy set was introduced by Atanassov.K.T [1] as a generalization of the notion of fuzzy set. Although a lot of studies have been done for fuzzy order structures the lattice-valued sets can be more appropriate to model natural problems. The justification to consider lattice valued fuzzy sets has been widely explained in the literature, since lattices are more richer structure and we can obtain non-comparable values of fuzzy sets. They can be applied, for instance, in image processing. Also the concepts of (α, β) cuts sets play a principal role in the relationship between fuzzy sets and crisp sets. They can be viewed as a bridge by which fuzzy sets and crisp sets are connected.

In the present paper, we have considered finite poset X and complete lattice L. The main purpose of this paper is to analyze the characterization of (α, β) cut sets of Intuitionistic L-fuzzy semi filter (ILFSF) of lattices

II. Preliminaries

In this section, some well-known definitions are recalled. It will be necessary in order to understand the new concepts and theorems introduced in this paper.

A poset is a nonempty set X with a partial order ≤. It is usually denoted as an ordered pair (X, ≤). A sub-poset of (X, ≤) is a subset Y of X in which the order is the one restricted from X, and usually denoted in the same way (≤). A semi filter on a poset X is any sub poset U, satisfying the following: for $x \in U$, $y \in X$, $x \le y$ implies $y \in U$. Dually, a semi ideal on X is any sub-poset D, satisfying: for $x \in D$, $y \in X$, $y \le x$ implies $y \in D$. A lattice is a poset L in which for each pair of elements x, y there is a greatest lower bound (glb, infimum, meet) and a least upper bound (lub, supremum, join), denoted respectively by $x \wedge y$ and $x \vee y$. These are binary operations on L. A non-empty poset L is said to be a complete lattice if infimum and supremum exist for each subset of L. Complete lattice possesses the top (1) and the bottom element (0). On the other hand, a lattice L is distributive, if each operation is distributive with respect to the other.

Definition 2.1. *Let (L,≤) be a complete lattice with least element 0 and greatest element 1 and an involutive order reversing operation $N : L \to L$. Then an Intuitionistic L−fuzzy subset (ILFS) A in a non-empty set X is defined as an object of the form $A = \{< x, \mu_A(x), \nu_A(x)> / x \in X\}$ where $\mu_A : X \to L$ is the degree of membership and $\nu_A : X \to L$ is the degree of non-membership of the element $x \in X$ satisfying $\mu_A(x) \le N(\nu_A(x))$.*

Definition 2.2. *An Intuitionistic L-fuzzy set of a Lattice is called as an Intuitionistic L-fuzzy semi filter whenever $x \le y$, we have $\mu_A(x) \le \mu_A(y)$ and $\nu_A(x) \ge \nu_A(y)$.*

Example 2.3. *Let X: {0,1,2,3} and L = {0,a,b,1}*

Define μ: X → L and ν: X → L as follows:

X	0	1	2	3
μ	a	a	b	1
ν	1	b	a	a

Here
$0 \le 1 \Rightarrow \mu_A(0) \le \mu_A(1)$ & $\nu_A(0) \ge \nu_A(1)$
$0 \le 2 \Rightarrow \mu_A(0) \le \mu_A(2)$ & $\nu_A(0) \ge \nu_A(2)$
$0 \le 3 \Rightarrow \mu_A(0) \le \mu_A(3)$ & $\nu_A(0) \ge \nu_A(3)$
$1 \le 2 \Rightarrow \mu_A(1) \le \mu_A(2)$ & $\nu_A(1) \ge \nu_A(2)$
$1 \le 3 \Rightarrow \mu_A(1) \le \mu_A(3)$ & $\nu_A(1) \ge \nu_A(3)$
$2 \le 3 \Rightarrow \mu_A(2) \le \mu_A(3)$ & $\nu_A(2) \ge \nu_A(3)$

Thus whenever $x \le y$, we have $\mu_A(x) \le \mu_A(y)$ and $\nu_A(x) \ge \nu_A(y)$.

Definition 2.4. An Intuitionistic L-fuzzy set of a Lattice is called as an Intuitionistic L-fuzzy semi ideal whenever $x \le y$, we have $\mu_A(x) \ge \mu_A(y)$ and $\nu_A(x) \le \nu_A(y)$.

*Corresponding author.

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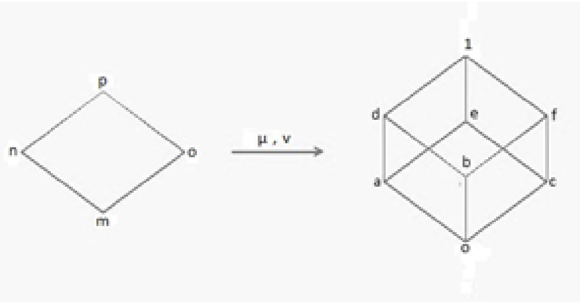
Definition 2.5. *Let A be an Intuitionistic L-fuzzy semi filter(ideal) on X. Then for α, β∈L, the (α, β) cut set of Intuitionistic L-fuzzy semi filter (ideal) is defined as the set* $A_{(\alpha,\beta)} = \{x \in X \,/\, \mu_A(x) \geq \alpha \text{ and } \nu_A(x) \leq \beta\}$.

Example 2.6.



μ	m	n	o	p	ν	m	n	o	p
μ_1	o	b	c	f	ν_1	f	c	b	o
μ_2	o	a	b	d	ν_2	d	b	a	o
μ_3	b	d	f	1	ν_3	1	f	d	b
μ_4	o	a	c	e	ν_4	e	c	a	o

Consider μ_1 and ν_1

If α ≤ β then	If α ≥ β then
A_{ob} = {o, p}	A_{bo} = {p}
A_{oc} = {n, p}	A_{co} = {p}
A_{bf} = {n, p}	A_{cb} = {o, p}
A_{cf} = {o, p}	A_{fc} = {p}
A_{of} = {m, n, o, p}	A_{fo} = {p}

Definition 2.7. *Let X and X' be any two sets. Let f: X → X' be any function and A be an Intuitionistic Fuzzy set in X, V be an Intuitionistic Fuzzy set in f(x) = X' defined by* $\mu_v(y) = \sup_{x \in f^{-1}(y)} \mu_A(x)$ *and* $\nu_v(y) = \inf_{x \in f^{-1}(y)} \nu_A(x)$ *for every x∈X and y∈X' then A is called a pre image of V under f and is denoted by* $f^{1}(V)$.

III. Properties of (α, β) Cut Sets

Let A be an Intuitionistic L fuzzy semi filter. Then the union and the intersection of two (α, β) cut sets is again a (α, β) cut set of A. The same can be proved in the case of Intuitionistic L fuzzy semi ideal also.

Proposition 3.1. *Let (X, ≤) be a poset. Let (L, ≤) be a complete lattice and A be an Intuitionistic L fuzzy set on X. Then the following are equivalent*

i) A is an Intuitionistic L-fuzzy semi filter
ii) $A_{(\alpha,\beta)}$ *is a crisp semi filter*

Proof. In the proof of this theorem we have to prove the equivalence among these conditions.
By combining the definition of Intuitionistic L-fuzzy semi filter and classical semi filter we can have the proof for the necessary part.

that is A = {< x, $\mu_A(x)$, $\nu_A(x)$> / x ∈ X} and if x ≤ y, we have $\mu_A(x) \leq \mu_A(y)$ and $\nu_A(x) \geq \nu_A(y)$.
Also $x \in A_{(\alpha,\beta)} \Rightarrow \mu_A(x) \geq \alpha$ and $\nu_A(x) \leq \beta$.
Hence $\alpha \leq \mu_A(x) \leq \mu_A(y)$ and $\nu_A(y) \geq \nu_A(x) \geq \beta$.
Therefore x ≤ y and y ∈ X ⇒ y∈ $A_{(\alpha,\beta)}$ which proves $A_{(\alpha,\beta)}$ is a crisp semi filter.

Since the elements of $A_{(\alpha,\beta)}$ are the elements of X we can easily prove the sufficient part.
Considering $\mu_A(x) = \alpha$ and $\nu_A(x) = \beta$ we will have $\mu_A(y) \geq \mu_A(x)$ and $\nu_A(y) \leq \nu_A(x)$.
⇒ whenever x ≤ y, we have $\mu_A(x) \leq \mu_A(y)$ and $\nu_A(x) \geq \nu_A(y)$.
Thus the proof of this theorem is an immediate consequence of the definitions of (α, β) cut sets and Intuitionistic L-fuzzy semi filter.

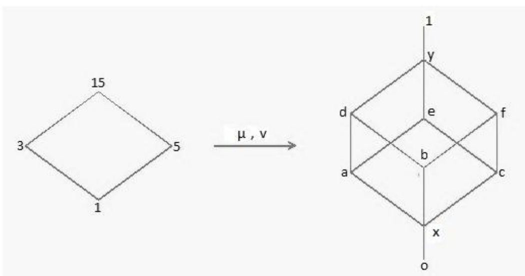
Proposition 3.2. *Let A and B are two Intuitionistic L fuzzy semi filters on X. If A ⊆ B then* $A_{(\alpha,\beta)} \subseteq B_{(\alpha,\beta)}$.

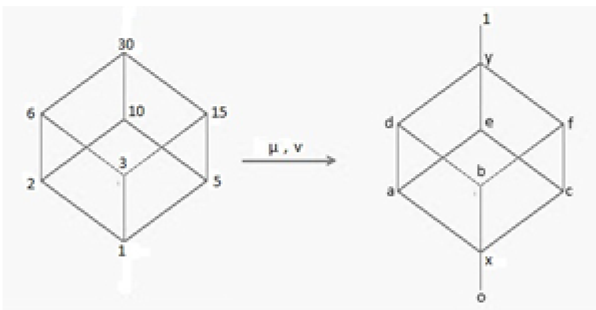
Proof. Taking an element x in $A_{(\alpha,\beta)}$, we will prove this x always belongs to $B_{(\alpha,\beta)}$. If A ⊆ B then $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$. From the definition of Intuitionistic L-fuzzy semi filter we have $x \leq y \Rightarrow \mu_A(x) \leq \mu_A(y)$ and $\nu_A(x) \geq \nu_A(y)$ and $x \in A_{(\alpha,\beta)} \Rightarrow \mu_A(x) \geq \alpha$ and $\nu_A(x) \leq \beta$.
Combining the above we have $\mu_B(x) \geq \alpha$ and $\nu_B(x) \leq \beta$ which implies $x \in B_{(\alpha,\beta)}$. Thus we have the result.

The converse of the above theorem is not true. That is $A_{(\alpha,\beta)} \subseteq B_{(\alpha,\beta)}$ does not imply A ⊆ B

Example 3.3.

Consider X = {1,2,3,5,6,10,15,30}.
A = {(1,x,y), (2,0,1), (3,b,e), (5,c,d), (6,0,1), (10,0,1), (15,f,x), (30,0,1)}
B = {(1,0,1), (2,a,f), (3,b,e), (5,c,d), (6,d,c), (10,e,b), (15,f,a), (30,1,0)}





Here $A_{(b, f)}$ = {3, 15} ana $B_{(b, f)}$ = {3, 6, 10, 15, 30}
clearly $A_{(b, f)} \subseteq B_{(b, f)}$, But $A \not\subset B$

Proposition 3.4. *The homomorpic image of an (α, β) cut set of an ILFSF is again a (α, β) cut set.*

Proof. Considering two lattices L_1 and L_2 and let $f : L_1 \to L_2$ be a homomorphism. Taking an Intuitionistic L fuzzy semi filter A of L_1 and let B = f(A) of L_2 we have B is an Intuitionistic L fuzzy semi filter of L_2 [3]. From the definition 2.7 if $A_{(\alpha, \beta)}$ is an (α, β) cut set of A, then $\mu_A(x) \geq \alpha$ and $\nu_A(x) \leq \beta$. Also from the definition 2.7 it is clear that $\mu_A(x) = \mu_B(f(x))$ and $\nu_A(x) = \nu_B(f(x))$. Hence $\mu_B(f(x)) \geq \alpha$ and $\nu_B(f(x)) \leq \beta$ which gives the result that $B_{(\alpha \beta)}$ is a (α, β) cut set of f(A).

Proposition 3.5. *The collection of all (α, β) cut sets of an Intuitionistic L-fuzzy semi filter is a complete lattice.*

Proof. Considering { $A_{(\alpha_i,\beta_i)}$ } be the family of (α, β) cut sets of an Intuitionistic L-fuzzy semi filter A, we can prove that this family is a complete lattice. First of all let us prove that { $A_{(\alpha_i,\beta_i)}$ } is a poset. For that we choose the partial order as $\subseteq$

i) cleary $A_{(\alpha_i,\beta_i)} \subseteq A_{(\alpha_i,\beta_i)}$ $\forall$i since $\alpha_i \leq \alpha_i$ and $\beta_i \leq \beta_i$ (Reflexivity)

ii) Let $A_{(\alpha_i,\beta_i)} \subseteq A_{(\alpha_j,\beta_j)}$ and $A_{(\alpha_j,\beta_j)} \subseteq A_{(\alpha_i,\beta_i)}$

$\Rightarrow \alpha_j \leq \alpha_i$ & $\beta_j \geq \beta_i$ and $\alpha_i \leq \alpha_j$ & $\beta_i \geq \beta_j$

$\Rightarrow \alpha_j \leq \alpha_i$ & $\alpha_i \leq \alpha_j$ and $\beta_j \geq \beta_i$ & $\beta_i \geq \beta_j$

$\Rightarrow \alpha_i = \alpha_j$ & $\beta_i = \beta_j$

Thus $A_{(\alpha_i,\beta_i)} = A_{(\alpha_j,\beta_j)}$ (Anti symmetry)

iii) Let $A_{(\alpha_i,\beta_i)} \subseteq A_{(\alpha_j,\beta_j)}$ and $A_{(\alpha_j,\beta_j)} \subseteq A_{(\alpha_k,\beta_k)}$

$\Rightarrow \alpha_j \leq \alpha_i,$ $\beta_j \geq \beta_i$ and $\alpha_k \leq \alpha_j$ & $\beta_k \geq \beta_j$

$\Rightarrow \alpha_i \geq \alpha_j \geq \alpha_k$ and $\beta_i \leq \beta_i \leq \beta_k$

$\Rightarrow \alpha_i \geq \alpha_k$ and $\beta_i \leq \beta_k$

Thus $A_{(\alpha_i,\beta_i)} \subseteq A_{(\alpha_k,\beta_k)}$ (Transitivity)

Hence { $A_{(\alpha_i,\beta_i)}$ } is poset

To prove: { $A_{(\alpha_i,\beta_i)}$ } is a complete lattice

Here every member of { $A_{(\alpha_i,\beta_i)}$ } is a subset of A formed by an (α, β) cut set

Considering the partial order $\subseteq$, we can have

$A_{(\alpha_i,\beta_i)} \vee A_{(\alpha_j,\beta_j)} = A_{(\alpha_i,\beta_i)} \cup A_{(\alpha_j,\beta_j)}$ and

$A_{(\alpha_i,\beta_i)} \wedge A_{(\alpha_j,\beta_j)} = A_{(\alpha_i,\beta_i)} \cap A_{(\alpha_j,\beta_j)}$

Clearly every 2-element subsets of { $A_{(\alpha_i,\beta_i)}$ } has least upper bound and greatest lower bound.

Hence { $A_{(\alpha_i,\beta_i)}$ } is a lattice.

Moreover every non empty subset of { $A_{(\alpha_i,\beta_i)}$ } also has both lub and glb

Thus { $A_{(\alpha_i,\beta_i)}$ } is a complete lattice.

Remark 3.6

- If the family of (α, β) cut sets is formed by all the classical semi filters of X then the membership and non membership functions μ and ν are order embedding.
- If α ≤ β then all the (α, β) cut sets of A will be the same irrespective of the membership and non membership functions.
- If α ≥ β then also we get the same (α, β) cut sets under any membership and non membership functions.
- It is clear that every $A_{(\alpha, \beta)}$ is closed under intersections and contains X.
- In example 3.3 we have that the family of (α, β) cut sets is formed by the classical semi filter of x. Also when we consider the other μ_i's and νi's there also we get the same sets. Moreover all the (α, β) cut sets are classical semi filters of X

IV. CONCLUSION

In this paper, we have considered the (α, β) cut sets of Intuitionistic L-fuzzy semi filter and investigated some of their useful properties. The relationships between (α, β) cut sets and the classical semi filters are also examined. Moreover, we have established necessary and sufficient conditions under which for a given family of classical semi filters , there exists an (α, β) cut set such that its collection of cuts coincides with the family of classical semi filters. In particular, we have done a deeper study for the Intuitionistic L-fuzzy semi filter such that its family of (α, β) cuts is formed by all the classical semi filters.

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