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A study on Q-intuitionistic L-fuzzy subsemiring of a semiring

*S. Sampathu**
Department of Mathematics,
Sri Muthukumaran College of Education,
Chennai, Tamil Nadu, India.
Email: sampathugokul@yahoo.in

S. Anita Shanthi
Department of Mathematics,
Annamalai University,
Tamil Nadu, India.
Email: shanthi.Anita@yahoo.com

A. Praveen Prakash
Department of Mathematics,
Hindustan University,
Chennai, Tamil Nadu, India.
Email: apraveenprakash@gmail.com

***Abstract*— In this paper, we made an attempt to study the algebraic nature of Q-intuitionistic L-fuzzy subsemiring of a semiring.**

Keywords- fuzzy subset, (Q, L)-fuzzy subset, (Q,L)-fuzzy subsemiring, Q-intuitionistic L-fuzzy subset, Q-intuitionistic L-fuzzy subsemiring, Q-intuitionistic L-fuzzy relation, Product of Q-intuitionistic L-fuzzy subsets.

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I. INTRODUCTION

After the introduction of fuzzy sets by L.A.Zadeh [25], several researchers explored on the generalization of the concept of fuzzy sets. The concept of intuitionistic fuzzy subset was introduced by K.T.Atanassov [5, 6], as a generalization of the notion of fuzzy set. The notion of fuzzy subnearrings and ideals was introduced by S.Abou Zaid [1]. A.Solairaju and R.Nagarajan [21, 22] have introduced and defined a new algebraic structure called Q-fuzzy subgroups. In this paper, we introduce the some theorems in Q-intuitionistic L-fuzzy subsemiring of a semiring and established some results.

II. PRELIMINARIES

Definition 2.1. [25] Let X be a non–empty set. A fuzzy subset A of X is a function A: $X \to [0, 1]$.

Definition 2.2. [21,22] *Let X be a non-empty set and L= (L, ≤) be a lattice with least element 0 and greatest element 1 and Q be a non-empty set. A (Q, L)-fuzzy subset A of X is a function A: $X\times Q \to L$.*

Definition 2.3. [18] *Let (R, +, ·) be a semiring and Q be a non empty set. A (Q, L)- fuzzy subset A of R is said to be a (Q, L)-fuzzy subsemiring (QLFSSR) of R if the following conditions are satisfied:*

(i) $A(x+y, q) \geq A(x, q) \wedge A(y, q)$,

(ii) $A(xy, q) \geq A(x, q) \wedge A(y, q)$, for all x and y in R and q in Q.

Example 2.4. Let (N , + , •) be a semiring and Q={p}, Then the (Q, L)-Fuzzy Set □ of N is defined by

$$A(x) = \begin{cases} 0.63 & \text{if x is even} \\ 0.27 & \text{if x is odd.} \end{cases}$$

Clearly A is an (Q, L)-Fuzzy subsemiring.

Definition 2.5. [5,6] *An intuitionistic fuzzy subset (IFS) A in X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$, where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.*

Definition 2.6. *Let (L, ≤) be a complete lattice with an involutive order reversing operation N: $L \to L$ and Q be a nonempty set. A Q-intuitionistic L-fuzzy subset (QILFS) A in X is defined as an object of the form $A=\{< (x, q), \mu_A(x, q), \nu_A(x, q) > / x$ in X and q in Q }, where $\mu_A : X\times Q \to L$ and $\nu_A : X\times Q \to L$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ satisfying $\mu_A(x) \leq N(\nu_A(x))$.*

Definition 2.7. *Let A and B be any two Q-intuitionistic L-fuzzy subsets of a set X. We define the following operations:*

i) $(A \cap B = \{ \langle x, \mu_A(x,q) \wedge \mu_B(x,q), \nu_A(x,q) \vee \nu_B(x,q) \rangle \}$, for all $x \in X$ and q in Q. (ii) $A \cup B = \{ \langle x, \mu_A(x,q) \vee \mu_B(x,q), \nu_A(x,q) \wedge \nu_B(x,q) \rangle \}$, for all $x \in X$ and q in Q. (iii) □A= $\{ \langle x, \mu_A(x,q), 1 - \mu_A(x,q) \rangle / x \in X \}$, for all x in X and q in Q. (iv) ◊A = $\{ \langle x, 1 - \nu_A(x,q), \nu_A(x,q) \rangle / x \in X \}$, for all x in X and q in Q.

Definition 2.8. *Let R be a semiring. A Q-intuitionistic L-fuzzy subset A of R is said to be a Q-intuitionistic L-fuzzy subsemiring (QILFSSR) of R if it satisfies the following conditions:*

(i)$\mu_A(x + y,q) \geq \mu_A(x,q) \wedge \mu_A(y,q)$, (ii)$\mu_A(xy,q) \geq \mu_A(x,q) \wedge \mu_A(y,q)$,(iii)$\nu_A(x + y,q) \leq \nu_A(x,q) \vee \nu_A(y,q)$,(iv)$\nu_A(xy,q) \leq \nu_A(x,q) \vee \nu_A(y,q)$, for all x and y in R and q in Q.

Definition 2.9. Let A and B be any two Q-intuitionistic L-fuzzy subsemiring of a semiring G and H, respectively. The

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*Corresponding author.

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product of A and B, denoted by A×B, is defined as $A\times B = \{ \langle (x,y),q), \mu_{AxB}((x,y),q), \nu_{AxB}((x,y),q) \rangle$ / for all x in G and y in H and q in Q }, where $\mu_{AxB}((x,y),q) = \mu_A(x,q)\wedge \mu_B(y,q)$ and $\nu_{AxB}((x, y),q) = \nu_A(x,q) \vee \nu_B(y,q)$.

Definition 2.10. *Let A be an Q-intuitionistic L-fuzzy subset in a set S, the strongest Q-intuitionistic L-fuzzy relation on S, that is a Q-intuitionistic L-fuzzy relation on A is V given by* $\mu_V((x,y),q) = \mu_A(x,q) \wedge \mu_A(y,q)$ *and* $\nu_V((x, y),q) = \nu_A(x,q)\vee \nu_A(y,q)$, *for all x and y in S and q in Q.*

Definition 2.11. *Let (R, +, •) and (R^{I}, +, •) be any two semirings. Let* $f : R \rightarrow R^{I}$ *be any function and A be an Q-intuitionistic L-fuzzy subsemiring in R, V be an Q-intuitionistic L-fuzzy subsemiring in* $f(R) = R^{I}$, *defined by* $\mu_v(y,q) = \sup_{x\in f^{-1}(y)} \mu_A(x,q)$ *and* $\nu_V(y,q) = \inf_{x \in f^{-1}(y)} \nu_A(x,q)$, *for all x in R and y in R^{I}. Then A is called a preimage of V under f and is denoted by* $f^{-1}(V)$.

Definition 2.12. *Let A be an Q-intuitionistic L-fuzzy subsemiring of a semiring (R, +, •) and a in R. Then the pseudo Q-intuitionistic L-fuzzy coset* $(aA)^p$ *is defined by* $((a\mu_A)^p)(x,q) = p(a)\mu_A(x,q)$ *and* $((a\nu_A)^p)(x,q) = p(a)\nu_A(x,q)$, *for every x in R and for some p in P and q in Q.*

III. PROPERTIES OF Q-INTUITIONISTIC L-FUZZY SUBSEMIRING OF A SEMIRING R

Theorem 3.1. *Intersection of any two Q-intuitionistic L-fuzzy subsemiring of a semiring R is a Q-intuitionistic L-fuzzy subsemiring of R.*

Proof. *Let A and B be any two Q-intuitionistic L-fuzzy subsemirings of a semiring R and x and y in R and q in Q. Let* $A=\{(x,q),\mu_A(x,q),\nu_A(x,q))/ \ x\in R$ *and q in Q} and* $B=\{(x,q),\mu_B(x,q),\nu_B(x,q))/x\in R$ *and q in Q} and also let* $C=A\cap B=\{(x,q),\mu_C(x,q),\ \nu_C(x,q))/x\in R$ *and q in Q},where* $\mu_A(x,q)\wedge\mu_B(x,q)=\mu_C(x,q)$ *and* $\nu_A(x,q)\vee\nu_B(x,q)= \nu_C(x,q)$. *Now,*$\mu_C(x+y,q)=\mu_A(x+y,q)\wedge\mu_B(x+y,q)\geq\{\mu_A(x,q)\wedge\mu_A(y,q)\}\wedge\{\mu_B(x,q)\wedge\mu_B(y,q)\}=\{\mu_A(x,q)\wedge\mu_B(x,q)\}\wedge\{\mu_A(y,q)\wedge\mu_B(y,q)\}=\mu_C(x,q)\wedge\mu_C(y,q)$.*Therefore,* $\mu_C(x+y,q) \geq \mu_C(x,q)\wedge\mu_C(y,q)$, *for all x and y in R and q in Q. And,* $\mu_C(xy,q) = \mu_A(xy,q)\wedge\mu_B(xy,q)\geq\{\mu_A(x,q)\wedge\mu_A(y,q)\}\wedge\{\mu_B(x,q)\wedge\mu_B(y,q)\}=\{\mu_A(x,q)\wedge\mu_B(x,q)\}\wedge\{\mu_A(y,q)\wedge\mu_B(y,q)\}\}=\mu_C(x,q)\wedge\mu_C(y,q)$. *Therefore,* $\mu_C(xy,q)\geq\mu_C(x,q)\wedge\mu_C(y,q)$, *for all x and y in R and q in Q. Now,* $\nu_C(x+y,q)=\nu_A(x+y,q)\vee\nu_B(x+y,q)\leq\{\nu_A(x,q)\vee\nu_A(y,q)\}\vee\{\nu_B(x,q)\vee\nu_B(y,q)\}=\{\nu_A(x,q)\vee\nu_B(x,q)\}\vee\{\nu_A(y,q)\vee\nu_B(y,q)\}=\nu_C(x,q)\vee\nu_C(y,q)$. *Therefore,* $\nu_C(x+y,q)\leq\nu_C(x,q)\vee \nu_C(y,q)$, *for all x and y in R and q in Q. And,* $\nu_C(xy,q)=\nu_A(xy,q)\vee\nu_B(xy,q)\leq \{\nu_A(x,q)\vee\nu_A(y,q)\}\vee\{ \nu_B(x,q)\vee \nu_B(y,q)\}\}=\{\nu_A(x,q)\vee \nu_B(x,q) \}\vee \{ \nu_A(y,q)\vee \nu_B(y,q)\}\} = \nu_C(x,q)\vee \nu_C(y,q)$.*Therefore,* $\nu_C(xy,q) \leq\nu_C(x,q)\vee \nu_C(y,q)$, *for all x and y in R and q in Q. Therefore C is a Q-intuitionistic L-fuzzy subsemiring of R. Hence the Q-intersection of any two Q-intuitionistic L-fuzzy subsemirings of a semiring R is an Q-intuitionistic L-fuzzy subsemiring of R.*

Theorem 3.2. *The intersection of a family of Q-intuitionistic L-fuzzy subsemirings of semiring R is a Q-intuitionistic L-fuzzy subsemiring of R.*

Proof. Let $\{V_i : i\in I\}$ be a family of Q-intuitionistic L-fuzzy subsemirings of a semiring R and let $A= \bigcap_{i\in I} Vi$. Let x and y in R and q in Q. Then, $\mu\ (x+y)= \inf_{i \in I} \mu_{Vi}\ (x+y) \geq \inf_{i \in I} \{\mu_{Vi}(x)\wedge\mu_{Vi}(y)\}= \inf_{i \in I} \mu_{Vi}(x)\wedge \inf_{i \in I} \mu_{Vi}(y) =\mu_A(x)\wedge\mu_A(y)$. Therefore, $\mu_A(x+y)\geq \mu_A(x) \wedge\mu_A(y)$, for all x and y in R and q in Q. And, $\mu_A(xy)= \inf_{i \in I} \mu_{Vi}(xy)\geq \inf_{i \in I} \{\mu_{Vi}(x)\wedge\mu_{Vi}(y)\}= \inf_{i \in I} \mu_{Vi}(x)\wedge \inf_{i \in I} \mu_{Vi}(y)=\mu_A(x)\wedge\mu_A(y)$. Therefore, $\mu_A(xy)\geq\mu_A(x)\wedge\mu_A(y)$, for all x and y in R and q in Q. Now, $\nu_A(x+y)= \sup_{i \in I} \nu_{Vi}(x+y)\leq \sup_{i \in I} \{\nu_{Vi}(x)\vee\nu_{Vi}(y)\}= \sup_{i \in I} \nu_{Vi}(x)\vee \sup_{i \in I} \nu_{Vi}(y)=\nu_A(x)\vee\nu_A(y)$. Therefore, $\nu_A(x+y)\leq\nu_A(x)\vee\nu_A(y)$, for all x and y in R and q in Q. And, $\nu_A(xy)= \sup_{i \in I} \nu_{Vi}(xy)\leq \sup_{i \in I} \{\nu_{Vi}(x)\vee\nu_{Vi}(y)\}= \sup_{i \in I} \nu_{Vi}(x)\vee \sup_{i \in I} \nu_{Vi}(y)=\nu_A(x)\vee\nu_A(y)$. Therefore, $\nu_A(xy)\leq \nu_A(x)\vee\nu_A(y)$, for all x and y in R. That is, A is a Q-intuitionistic L-fuzzy subsemiring of a semiring R. Hence, the Q-intersection of a family of Q-intuitionistic L-fuzzy subsemirings of R is a Q-intuitionistic L-fuzzy subsemiring of R.

Theorem 3.3. *If A and B are any two Q-intuitionistic L-fuzzy subsemirings of the semirings* R_1 *and* R_2 *respectively, then* $A\times B$ *is a Q-intuitionistic L-fuzzy subsemiring of* $R_1\times R_2$.

Proof. Let A and B be two Q-intuitionistic L-fuzzy subsemirings of the semirings R_1 and R_2 respectively. Let x_1 and x_2 be in R_1, y_1 and y_2 be in R_2 and q in Q. Then (x_1,y_1) and (x_2,y_2) are in $R_1\times R_2$. Now, $\mu_{A\times B}[((x_1,y_1)+(x_2,y_2)),q]=\mu_{A\times B}((x_1+x_2,y_1+y_2),q)=\mu_A((x_1+x_2),q)\wedge \mu_B((y_1+y_2),q)\geq\{\{\mu_A(x_1,q)\wedge\mu_A(x_2,q)\}\wedge\{\mu_B(y_1,q)\wedge\mu_B(y_2,q)\}\} =\{\{\mu_A(x_1,q)\wedge\mu_B(y_1,q)\}\wedge\{\mu_A(x_2,q)\wedge\mu_B(y_2,q)\}\}=\mu_{A\times B}((x_1,y_1),q)\wedge \mu_{A\times B}((x_2,y_2),q)$.Therefore,$\mu_{A\times B}[((x_1,y_1)+(x_2,y_2)),q]\geq\mu_{A\times B}((x_1,y_1),q)\wedge\mu_{A\times B}((x_2,y_2),q)$.Also,$\mu_{A\times B}[((x_1,y_1)(x_2,y_2)),q)]=\mu_{A\times B}((x_1x_2,y_1y_2),q)=\mu_A(x_1x_2,q)\wedge\mu_B(y_1y_2,q)\geq\{\{\mu_A(x_1,q)\wedge\mu_A(x_2,q)\}\wedge\{\mu_B(y_1,q)\wedge\mu_B(y_2,q)\}\}=\{\{\mu_A(x_1,q)\wedge\mu_B(y_1,q)\}\wedge\{\mu_A(x_2,q)\wedge\mu_B(y_2,q)\}\} =\mu_{A\times B}((x_1,y_1),q)\wedge\mu_{A\times B}((x_2,y_2),q)$.Therefore,$\mu_{A\times B}[((x_1,y_1)(x_2,y_2)),q]\geq\mu_{A\times B}((x_1,y_1),q)\wedge\mu_{A\times B}((x_2,y_2),q)$.
Now,$\nu_{A\times B}[((x_1,y_1)+(x_2,y_2),q)]=\nu_{A\times B}((x_1+x_2,y_1+y_2),q)= \nu_A((x_1+x_2),q)\vee\nu_B((y_1+y_2),q)\leq\{\{\nu_A(x_1,q)\vee\nu_A(x_2,q)\}\vee\{\nu_B(y_1,q)\vee \nu_B(y_2,q)\}\}=\{\{\nu_A(x_1,q)\vee\nu_B(y_1,q)\}\vee\{\nu_A(x_2,q)\vee\nu_B(y_2,q)\}\} =\nu_{A\times B}((x_1,y_1),q)\vee\nu_{A\times B}((x_2,y_2),q)$.Therefore, $\nu_{A\times B}[((x_1,y_1)+(x_2,y_2),q)]\leq\nu_{A\times B}((x_1,y_1),q)\vee\nu_{A\times B}((x_2,y_2),q)$.
Also,$\nu_{A\times B}[(x_1,y_1)(x_2,y_2),q]=\nu_{A\times B}((x_1x_2,q)(y_1y_2,q))=\nu_A(x_1x_2,q)\vee \nu_B(y_1y_2,q)\leq\{\nu_A(x_1,q)\vee\nu_A(x_2,q)\}\vee\{\nu_B(y_1,q)\vee\nu_B(y2,q)\}\} =\{\{\nu_A(x_1,q)\vee\nu_B(y_1,q)\}\vee\{\nu_A(x_2,q)\vee\nu_B(y_2,q)\}= \nu_{A\times B}\ ((x_1,y_1),q)\vee \nu_{A\times B}\ ((x_2,y_2),q)$. Therefore, $\nu_{A\times B}\ [((x_1,y_1)(x_2,y_2)),q]\leq \nu_{A\times B}\ ((x_1,\ y_1),\ q)\ \vee\ \nu_{A\times B}\ (\ (x_2,\ y_2),q)$. Hence A×B is a Q-intuitionistic L-fuzzy subsemiring of semiring of $R_1\times R_2$.

Theorem 3.4. *Let A be a Q-intuitionistic L-fuzzy subset of a semiring R and V be the strongest Q-intuitionistic L-fuzzy relation of R. Then A is a Q-intuitionistic L-fuzzy subsemiring of R if and only if V is a Q-intuitionistic L-fuzzy subsemiring of R×R.*

Proof. Suppose that A is a Q-intuitionistic L-fuzzy subsemiring of a semiring R. Then for any $x=(x_1,x_2)$ and $y=(y_1,y_2)$ are in R×R and q in Q . We have, $\mu_V((x+y),q) =\mu_V[((x_1,x_2)+(y_1,y_2)),q)]=\mu_V((x_1+y_1,x_2+y_2),q)=\mu_A((x_1+y_1),q)\wedge \mu_A((x_2+y_2),q)\geq\{\{\mu_A(x_1,q)\wedge\mu_A(y_1,q)\}\wedge\{\mu_A(x_2,q)\wedge\mu_A(y_2,q)\}\}= \{\{\mu_A(x_1,q)\wedge\mu_A(x_2,q)\}\wedge\{\mu_A(y_1,q\ )\wedge\mu_A(y_2,q)\}\}= \mu_V((x_1,x_2),q)\wedge\mu_V\ ((y_1,\ y_2),q)=\mu_V\ (x,q)\wedge\mu_V(y,q)$. Therefore, $\mu_V\ ((x+y),q)\geq\mu_V(x,q)\wedge\mu_V\ (y,q)$, for all x and y in R×R and q in Q.And,$\mu_V(xy,q)=\mu_V[((x_1,x_2)(y_1,y_2),q)]=\mu_V((x_1y_1,x_2y_2),q)= \mu_A(x_1y_1,q)\wedge\mu_A(x_2y_2,q)\geq\{\{\mu_A(x_1,q)\wedge\mu_A(y_1,q)\}\wedge\{\mu_A(x_2,q)\wedge \mu_A(y_2,q)\}\}=\{\{\mu_A(x_1,q)\wedge\mu_A(x_2,q)\}\wedge\{\mu_A(y_1,q)\wedge\mu_A(y_2,q)\}\}= \{\mu_V((x_1,x_2),q)\wedge\mu_V((y_1,y_2),q)\}=\{\mu_V(x,q)\wedge\mu_V(y,q)\}$.Therefore, $\mu_V(xy,q)\geq\mu_V(x,q)\wedge\mu_V(y,q)$, for all x and y in R×R and q in Q.

We have, $\nu_V((x+y),q)=\nu_V[((x_1,x_2)+(y_1,y_2),q)]=\nu_V((x_1+y_1, x_2+y_2),q)=\nu_A((x_1+y_1),q)\vee\nu_A((x_2+y_2),q)\}\le\{\{\nu_A(x_1,q)\vee\nu_A(y_1,q)\}\vee\{\nu_A(x_2,q)\vee\nu_A(y_2,q)\}\}=\{\{\nu_A(x_1,q)\vee\nu_A(x_2,q)\}\vee\{\nu_A(y_1,q)\vee\nu_A(y_2,q)\}\}=\{\nu_V((x_1,x_2),q)\vee\nu_V((y_1,y_2),q)\}=\nu_V(x,q)\vee\nu_V(y,q)$. Therefore, $\nu_V((x+y),q)\le\nu_V(x,q)\vee\nu_V(y,q)$, for all x and y in R×R and q in Q. And, $\nu_V(xy,q)=\nu_V[((x_1,x_2)(y_1,y_2),q)]=\nu_V((x_1y_1,x_2y_2),q)=\nu_A(x_1y_1,q)\vee\nu_A(x_2y_2,q)\le\{\{\nu_A(x_1,q)\vee\nu_A(y_1,q)\}\vee\{\nu_A(x_2,q),\nu_A(y_2,q)\}\}=\{\{\nu_A(x_1,q)\vee\nu_A(x_2,q)\}\vee\{\nu_A(y_1,q)\vee\nu_A(y_2,q)\}\}=\nu_V((x_1,x_2),q)\vee\nu_V((y_1,y_2),q)=\nu_V(x,q)\vee\nu_V(y,q)$. Therefore, $\nu_V(xy,q)\le\nu_V(x,q)\vee\nu_V(y,q)$, for all x and y in R×R and q in Q. This proves that V is a Q-intuitionistic L-fuzzy subsemiring of R×R. Conversely assume that V is a Q-intuitionistic L-fuzzy subsemiring of R×R, then for any $x=(x_1,x_2)$ and $y=(y_1,y_2)$ are in R×R and q in Q, we have $\mu_A((x_1+y_1),q)\wedge\mu_A((x_2+y_2),q)=\mu_V((x_1+y_1,x_2+y_2),q)=\mu_V[((x_1,x_2),q)+((y_1,y_2),q)]=\mu_V((x+y),q)\ge\mu_V(x,q)\wedge\mu_V(y,q)=\mu_V((x_1,x_2),q)\wedge\mu_V((y_1,y_2),q)=\{\{\mu_A(x_1,q)\wedge\mu_A(x_2,q)\}\wedge\{\mu_A(y_1,q),\mu_A(y_2,q)\}\}$. If $\mu_A((x_1+y_1),q)\le\mu_A((x_2+y_2),q)$, $\mu_A(x_1,q)\le\mu_A(x_2,q)$, $\mu_A(y_1,q)\le\mu_A(y_2,q)$, we get, $\mu_A((x_1+y_1),q)\ge\mu_A(x_1,q)\wedge\mu_A(y_1,q)$, for all x_1 and y_1 in R and q in Q. And, $\mu_A(x_1y_1),q)\wedge\mu_A(x_2y_2),q)\}=\mu_V((x_1y_1,x_2y_2),q)=\mu_V[((x_1,x_2)(y_1,y_2),q)]=\mu_V(xy,q)\ge\mu_V(x,q)\wedge\mu_V(y,q)=\mu_V((x_1,x_2),q)\wedge\mu_V((y_1,y_2),q)\}=\{\{\mu_A(x_1,q)\wedge\mu_A(x_2,q)\}\wedge\{\mu_A(y_1,q)\wedge\mu_A(y_2,q)\}\}$. If $\mu_A(x_1y_1,q)\le\mu_A(x_2y_2,q)$, $\mu_A(x_1,q)\le\mu_A(x_2,q)$, $\mu_A(y_1,q)\le\mu_A(y_2,q)$, we get $\mu_A(x_1y_1,q)\ge\mu_A(x_1,q)\wedge\mu_A(y_1,q)$, for all x_1 and y_1 in R and q in Q. We have $\nu_A((x_1+y_1),q)\vee\nu_A((x_2+y_2),q)=\nu_V((x_1+y_1,x_2+y_2),q)=\nu_V[((x_1,x_2)+(y_1,y_2),q)]=\nu_V(x+y,q)\le\nu_V(x,q)\vee\nu_V(y,q)=\nu_V((x_1,x_2),q)\vee\nu_V((y_1,y_2),q)=\{\{\nu_A(x_1,q)\vee\nu_A(x_2,q)\}\vee\{\nu_A(y_1,q)\vee\nu_A(y_2,q)\}\}$. If $\nu_A(x_1+y_1,q)\ge\nu_A(x_2+y_2,q)$, $\nu_A(x_1,q)\ge\nu_A(x_2,q)$, $\nu_A(y_1,q)\ge\nu_A(y_2,q)$, we get, $\nu_A(x_1+y_1,q)\le\nu_A(x_1,q)\vee\nu_A(y_1,q)$, for all x_1 and y_1 in R and q in Q. And, $\nu_A(x_1y_1,q)\vee\nu_A(x_2y_2,q)=\nu_V((x_1y_1,x_2y_2),q)=\nu_V[((x_1,x_2),(y_1,y_2)),q)]=\nu_V(xy,q)\le\nu_V(x,q)\vee\nu_V(y,q)=\nu_V((x_1,x_2),q)\vee\nu_V((y_1,y_2),q)=\{\{\nu_A(x_1,q)\vee\nu_A(x_2,q)\}\vee\{\nu_A(y_1,q),\nu_A(y_2,q)\}\}$. If $\nu_A(x_1y_1,q)\ge\nu_A(x_2y_2,q)$, $\nu_A(x_1,q)\ge\nu_A(x_2,q)$, $\nu_A(y_1,q)\ge\nu_A(y_2,q)$, we get $\nu_A(x_1y_1,q)\le\nu_A(x_1,q)\vee\nu_A(y_1,q)$, for all x_1 and y_1 in R and q in Q. Therefore A is a Q-intuitionistic L-fuzzy subsemiring of R.

Theorem 3.5. *If A is a Q-intuitionistic L-fuzzy subsemiring of a semiring (R, +,•), then $H=\{x/x\in R: \mu_A(x,q)=1, \nu_A(x,q)=0\}$ is either empty or is a subsemiring of R.*

Proof. If no element satisfies this condition, then H is empty. If x and y in H and q in Q, then $\mu_A(x+y,q)\ge\mu_A(x,q)\wedge\mu_A(y,q)=1\wedge1=1$. Therefore, $\mu_A(x+y,q)=1$. And $\mu_A(xy,q)\ge\mu_A(x,q)\wedge\mu_A(y,q)=1\wedge1=1$. Therefore, $\mu_A(xy,q)=1$. Now, $\nu_A(x+y,q)\le\nu_A(x,q)\vee\nu_A(y,q)=0\vee0=0$. Therefore, $\nu_A(x+y,q)=0$. And $\nu_A(xy,q)\le\nu_A(x,q)\vee\nu_A(y,q)=0\vee0=0$. Therefore, $\nu_A(xy,q)=0$. We get x+y, xy in H. Therefore, H is a subsemiring of R. Hence H is either empty or is a subsemiring of R.

Theorem 3.6. *If A be a Q-intuitionistic L-fuzzy subsemiring of a semiring (R, +, •), then (i) if $\mu_A(x+y,q)=0$, then either $\mu_A(x,q)=0$ or $\mu_A(y,q)=0$, for all x and y in R and q in Q. (ii) if $\mu_A(x+y,q)=1$, then either $\mu_A(x,q)=1$ or $\mu_A(y,q)=1$, for all x and y in R and q in Q.*

Proof. Let x and y in R and q in Q. (i) By the definition $\mu_A(x+y,q)\ge\mu_A(x)\wedge\mu_A(y)$, which implies that $0\ge\mu_A(x,q)\wedge\mu_A(y,q)$. Therefore, either $\mu_A(x,q)=0$ or $\mu_A(y,q)=0$. (ii) By the definition $\mu_A(x+y,q)\le\mu_A(x,q)\vee\mu_A(y,q)$, which implies that $1\le\mu_A(x,q)\vee\mu_A(y,q)$. Therefore, either $\mu_A(x,q)=1$ or $\mu_A(y,q)=1$.

Theorem 3.7. *If A is a Q-intuitionistic L-fuzzy subsemiring of a semiring (R, +, •), then $H=\{\langle(x,q),\mu_A(x,q)\rangle: 0<\mu_A(x,q)\le1$ and $\nu_A(x,q)=0\}$ is either empty or is a subsemiring of R.*

Proof. If no element satisfies this condition, then H is empty. If x and y satisfies this condition, then $\nu_A(x+y,q)\le\nu_A(x,q)\vee\nu_A(y,q)=0\vee0=0$. Therefore, $\nu_A(x+y,q)=0$, for all x and y in R and q in Q. And, $\nu_A(xy,q)\le\nu_A(x,q)\vee\nu_A(y,q)=0\vee0=0$. Therefore, $\nu_A(xy,q)=0$, for all x and y in R and q in Q. And, $\mu_A(x+y,q)\ge\mu_A(x,q)\wedge\mu_A(y,q)$. Therefore, $\mu_A(x+y,q)\ge\mu_A(x,q)\wedge\mu_A(y,q)$, for all x and y in R and q in Q. And, $\mu_A(xy,q)\ge\mu_A(x,q)\wedge\mu_A(y,q)$. Therefore, $\mu_A(xy,q)\ge\mu_A(x,q)\wedge\mu_A(y,q)$, for all x and y in R and q in Q. Hence H is a fuzzy subsemiring of R. Therefore, H is either empty or is a subsemiring of R.

Theorem 3.8. *If A is a Q-intuitionistic L-fuzzy subsemiring of a semiring (R, +, •) then $H=\{\langle(x,q),\mu_A(x,q)\rangle: 0<\mu_A(x,q)\le1\}$ is either empty or an fuzzy subsemiring of R.*

Proof. *If no element satisfies this condition, then H is empty. If x and y satisfies this condition, then $\mu_A(x+y,q)\ge\mu_A(x,q)\wedge\mu_A(y,q)$.*

Therefore, $\mu_A(x+y,q)\ge\mu_A(x,q)\wedge\mu_A(y,q)$, for all x and y in R and q in Q. And $\mu_A(xy,q)\ge\mu_A(x,q)\wedge\mu_A(y,q)$. Therefore, $\mu_A(xy,q)\ge\mu_A(x,q)\wedge\mu_A(y,q)$, for all x and y in R and q in Q. Therefore, H is either empty or is a subsemiring of R.

Theorem 3.9. *If A is a Q-intuitionistic L-fuzzy subsemiring of a semiring (R, +, •), then $H=\{\langle(x,q),\nu_A(x,q)\rangle: 0<\nu_A(x,q)\le1\}$ is either empty or is a subsemiring of R.*

Proof. If no element satisfies this condition, then H is empty. If x and y satisfies this condition, then $\nu_A(x+y,q)\le\nu_A(x,q)\vee\nu_A(y,q)$.

Therefore, $\nu_A(x+y,q)\le\nu_A(x,q)\vee\nu_A(y,q)$, for all x and y in R and q in Q. And $\nu_A(xy,q)\le\nu_A(x,q)\vee\nu_A(y,q)$. Therefore, $\nu_A(xy,q)\le\nu_A(x,q)\vee\nu_A(y,q)$, for all x and y in R and q in Q. Hence H is either empty or is a subsemiring of R.

Theorem 3.10. *If A is a Q-intuitionistic L-fuzzy subsemiring of a semiring (R, +,•), then A is a Q-intuitionistic L-fuzzy subsemiring of R.*

Proof. Let A be a Q-intuitionistic L-fuzzy subsemiring of a semiring R. Consider $A=\{\langle(x,q),\mu_A(x,q),\nu_A(x,q)\rangle\}$, for all x in R and q in Q, we take $\square A=B=\{\langle(x,q),\mu_B(x,q),\nu_B(x,q)\rangle\}$, where $\mu_B(x,q)=\mu_A(x,q)$, $\nu_B(x,q)=1-\mu_A(x,q)$. Clearly, $\mu_B(x+y,q)\ge\mu_B(x,q)\wedge\mu_B(y,q)$, for all x and y in R and q in Q. Also $\mu_B(xy,q)\ge\mu_B(x,q)\wedge\mu_B(y,q)$, for all x and y in R. Since A is an Q-intuitionistic L-fuzzy subsemiring of R, we have $\mu_A(x+y,q)\ge\mu_A(x,q)\wedge\mu_A(y,q)$, for all x and y in R, which implies that $1-\nu_B(x+y,q)\ge\{(1-\nu_B(x,q))\wedge(1-\nu_B(y,q))\}$, which implies that $\nu_B(x+y,q)\le1-\{(1-\nu_B(x,q))\wedge(1-\nu_B(y,q))\}=\nu_B(x,q)\vee\nu_B(y,q)$. Therefore, $\nu_B(x+y,q)\le\nu_B(x,q)\vee\nu_B(y,q)$, for all x and y in R and q in Q. And $\mu_A(xy,q)\ge\mu_A(x,q)\wedge\mu_A(y,q)$, for all x and y in R and q in Q, which implies that $1-\nu_B(xy,q)\ge\{(1-\nu_B(x,q))\wedge(1-\nu_B(y,q))\}$ which implies that $\nu_B(xy,q)\le1-\{(1-\nu_B(x,q))\wedge(1-\nu_B(y,q))\}=\nu_B(x,q)\vee\nu_B(y,q)$. Therefore, $\nu_B(xy,q)\le\nu_B(x,q)\vee\nu_B(y,q)$, for all x and y in R and q in Q. Hence $B=\square A$ is a Q-intuitionistic L-fuzzy subsemiring of a semiring R.

Theorem 3.11. *If A is a Q-intuitionistic L-fuzzy subsemiring of a semiring (R, +, •), then ◊A is a Q-intuitionistic L-fuzzy subsemiring of R.*

Proof. Let A be a Q-intuitionistic L-fuzzy subsemiring of a semiring R. That is $A=\{\langle (x,q),\mu_A(x,q),\nu_A(x,q)\rangle\}$, for all x in R and q in Q. Let $\Diamond A=B=\{\langle (x,q),\mu_B(x,q), \nu_B(x,q)\rangle\}$, where $\mu_B(x,q)=1-\nu_A(x,q),\nu_B(x,q)=\nu_A(x,q)$.

Clearly $\nu_B(x+y,q)\leq\nu_B(x)\vee\nu_B(y)$, for all x and y in R and $\nu_B(xy,q)\leq\nu_B(x,q)\vee\nu_B(y,q)$, for all x and y in R and q in Q. Since A is a Q-intuitionistic L-fuzzy subsemiring of R, we have $\nu_A(x+y,q)\leq \nu_A(x,q)\vee \nu_A(y,q)$, for all x and y in R and q in Q, which implies that $1-\mu_B(x+y,q)\leq\{(1-\mu_B(x,q))\vee(1-\mu_B(y,q))\}$,which implies that $\mu_B(x+y,q)\geq 1-\{(1-\mu_B(x,q))\vee(1-\mu_B(y,q))\}=\mu_B(x,q)\wedge\mu_B(y,q)$.Therefore, $\mu_B(x+y,q)\geq\mu_B(x,q)\wedge\mu_B(y,q)$, for all x and y in R. And $\nu_A(xy,q)\leq \nu_A(x,q)\vee\nu_A(y,q)$, for all x and y in R and q in Q, which implies that $1-\mu_B(xy,q)\leq\{(1-\mu_B(x,q))\vee(1-\mu_B(y,q))\}$, which implies that $\mu_B(xy,q)\geq 1-\{(1-\mu_B(x,q))\vee(1-\mu_B(y,q))\}=\mu_B(x,q)\wedge\mu_B(y,q)$.
Therefore,$\mu_B(xy,q)\geq\mu_B(x,q)\wedge\mu_B(y,q)$, for all x and y in R and q in Q. Hence $B = \Diamond A$ is a Q-intuitionistic L-fuzzy subsemiring of a semiring R.

Theorem 3.12. Let A be a Q-intuitionistic L-fuzzy subsemiring of a semiring H and f is an isomorphism from a semiring R onto H. Then $A\circ f$ is a Q-intuitionistic L-fuzzy subsemiring of R.

Proof. Let x and y in R and q in Q, A be a Q-intuitionistic L-fuzzy subsemiring of a semiring H. Then we have, $(\mu_A\circ f)(x+y,q)=\mu_A(f(x+y),q)=\mu_A(f(x,q)+f(y,q))\geq\mu_A(f(x,q))\wedge\mu_A(f(y,q))\geq(\mu_A\circ f)(x,q)\wedge(\mu_A\circ f)(y,q)$, which implies that

$(\mu_A\circ f)(x+y,q)\geq(\mu_A\circ f)(x,q)\wedge(\mu_A\circ f)(y,q)$.
And$(\mu_A\circ f)(xy,q)=\mu_A(f(xy,q))=\mu_A(f(x,q)f(y,q))\geq\mu_A(f(x,q))\wedge\mu_A(f(y,q))\geq(\mu_A\circ f)(x,q)\wedge(\mu_A\circ f)(y,q)$,which implies that $(\mu_A\circ f)(xy,q)\geq(\mu_A\circ f)(x,q)\wedge(\mu_A\circ f)(y,q)$. Then we have, $(\nu_A\circ f)(x+y,q)=\nu_A(f(x+y,q))=\nu_A(f(x,q)+f(y,q))\leq\nu_A(f(x,q))\vee\nu_A(f(y,q)) \leq (\nu_A\circ f)(x,q)\vee(\nu_A\circ f)(y,q)$, which implies that $(\nu_A\circ f)(x+y,q)\leq(\nu_A\circ f)(x,q)\vee (\nu_A\circ f)(y,q)$.
And$(\nu_A\circ f)(xy,q)=\nu_A(f(xy,q))$
$=\nu_A(f(x,q)f(y,q))\leq\nu_A(f(x,q))\vee\nu_A(f(y,q)) \leq (\nu_A\circ f)(x,q) \vee (\nu_A\circ f)(y,q)$, which implies that $(\nu_A\circ f)(xy,q)\leq(\nu_A\circ f)(x,q)\vee(\nu_A\circ f)(y,q)$. Therefore $(A\circ f)$ is a Q-intuitionistic L-fuzzy subsemiring of a semiring R.

Theorem 3.13. *Let A be a Q-intuitionistic L-fuzzy subsemiring of a semiring (R, +, •), then the pseudo Q-intuitionistic L-fuzzy coset $(aA)^p$ is a Q-intuitionistic L- fuzzy subsemiring of a semiring R, for every a in R.*

Proof. Let A be a Q-intuitionistic L-fuzzy subsemiring of a semiring R. For every x and y in R and q in Q, we have, $((a\mu_A)^p)(x+y,q)=p(a)\mu_A(x+y,q)\geq p(a)\{(\mu_A(x,q)\wedge\mu_A(y,q)\}=p(a)\mu_A(x,q)\wedge p(a)\mu_A(y,q)=((a\mu_A)^p)(x,q)\wedge((a\mu_A)^p)(y,q)$.Therefore, $((a\mu_A)^p)(x+y,q)\geq((a\mu_A)^p)(x,q)\wedge((a\mu_A)^p)(y,q)$.Now,$((a\mu_A)^p)(xy,q)=p(a)\mu_A(xy,q)\geq p(a)\{\mu_A(x,q)\wedge\mu_A(y,q)\}=p(a)\mu_A(x,q)\wedge p(a)\mu_A(y,q)=((a\mu_A)^p)(x,q)\wedge((a\mu_A)^p)(y,q)$.Therefore, $((a\mu_A)^p)(xy,q)\geq((a\mu_A)^p)(x,q)\wedge((a\mu_A)^p)(y,q)$. For every x and y in R and q in Q, we have, $((a\nu_A)^p)(x+y,q) = p(a)\nu_A(x+y,q) \leq p(a)\{(\nu_A(x,q)\vee\nu_A(y,q)\}=p(a)\nu_A(x,q)\vee p(a)\nu_A(y,q)=((a\nu_A)^p)(x,q)\vee((a\nu_A)^p)(y,q)$.Therefore,$((a\nu_A)^p)(x+y,q)\leq((a\nu_A)^p)(x,q)\vee$

$((a\nu_A)^p)(y,q)$.Now,$((a\nu_A)^p)(xy,q)=p(a)\nu_A(xy,q)\leq p(a)\{\nu_A(x,q)\vee\nu_A(y,q)\}=p(a)\nu_A(x,q)\vee p(a)\nu_A(y,q)=((a\nu_A)^p)(x,q)\vee((a\nu_A)^p)(y,q)$.
Therefore,$((a\nu_A)^p)(xy,q)\leq((a\nu_A)^p)(x,q)\vee((a\nu_A)^p)(y,q)$.
Hence $(aA)^p$ is a Q-intuitionistic L-fuzzy subsemiring of a semiring R.

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