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M/M/1/K/N interdependent retrial queueing model with controllable arrival rates, balking and retention of reneged customers

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***Abstract*—In this paper, a finite capacity single server finite population interdependent retrial queueing model with controllable arrival rates, balking and retention of reneged customers is considered. The steady state solutions and the system characteristics are derived and analyzed for this model. Some particular cases of the model have been discussed. This model may be of great importance to the business facing the serious problem of customer impatience. Numerical results are given for better understanding and relevant conclusion is presented**

Keywords*— *retrial queue; reneging; customer retention; balking; interdependent primary arrival and service processes; finite capacity.

I. INTRODUCTION

Retrial queues have been widely used to model many problems in telephone switching systems, telecommunication networks and computer systems for competing to gain service from a central processing unit and so on. Moreover, retrial queues are also used as mathematical models for several computer systems: packet switching networks, shared bus local area networks operating under the carrier-sense multiple access protocol and collision avoidance star local area networks, etc. Retrial queueing systems are characterized by the feature that a blocked customer (a customer who finds the server unavailable) may leave the service area temporarily and join a retrial group in order to retry his request after some random time. Abandonment happens when a subscriber's call becomes rejected and the subscriber gets impatient and gives up after a certain time without getting service. The model studied in this paper not only takes into account retrials due to congestion but also considers the effects of balking and retention of reneged customers discipline.

The detailed account on retrial queues can be found in the book on retrial queues by Falin and Templeton [10]. Artalejo [2] analyzed queueing system with returning customers and waiting line. Artalejo [3] studied retrial queues with a finite number of sources. An extensive survey on retrial queues can be found in notable survey articles by Artalejo [4-6] and Kumar and Kumar Sharma [12-15] have studied the M/M/1/N queue with the concept of retention of reneged customers. Thiagarajan and Srinivasan [16] have analyzed M/M/C/K/N interdependent queueing model with controllable arrival rates balking, reneging and spares. Jain and Bhagat [11] have considered the finite population retrial queueing model with threshold recovery geometric arrivals and impatient customers. Recently Antline Nisha and Thiagarajan [17] have studied M/M/1/K/N interdependent retrial queueing model with controllable arrival rates. In general it is assumed that the arrival stream of primary calls, the service times and retrial times are mutually independent. But the primary arrival and service processes are interdependent in practical situations. Although it is natural in the real world, there are only few works taking into consideration retrial phenomena involving the interdependent controllable arrival rates.

In this paper, the M/M/1/K/N interdependent retrial queueing model with controllable arrival rates, balking and retention of reneged customers is considered. In section 2, the description of the model is given stating the relevant postulates. In section 3, the steady state equations are obtained. In section 4, the characteristics of the model are derived. In section 5, numerical results are calculated.

II. DESCRIPTION OF THE MODEL

Consider a single server finite capacity finite source retrial queueing system in which primary customers arrive according to the Poisson flow of rate λ_0 and λ_1, service times are exponentially distributed with rate μ. If a primary customer finds some server free, he instantly occupies it and leaves the system after service. Otherwise, if the server is busy, at the time of arrival of a primary call then with probability $H_1 \geq 0$ the arriving customer enters an orbit and repeats his demand after an exponential time with rate θ. Thus the Poisson flow of repeated call follow the retrial policy where the repetition times of each customer is assumed to be independent and exponentially distributed.

If an incoming repeated call finds the line free, it is served and leaves the system after service, while the source which produced this repeated call disappears. Otherwise, if the server is occupied at the time of a repeated call arrival with probability $(1\text{-}H_2)$ the source leaves the system

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without service. Each customer upon arriving in the queue will wait a certain length of time (reneging time) for his service to begin. If it has not begun by then, he will get impatient and may leave the queue without getting service with probability p and may remain in the queue for his service with probability (q = 1-p). The reneging times follow exponential distribution with parameter α.

It is assumed that the primary arrival process $[X_1(t)]$ and the service process $[X_2(t)]$ of the systems are correlated and follow a bivariate Poisson process given by

$$P(X_1 = x_1, X_2 = x_2; t)$$

$$= e^{-(\lambda_i + \mu - \varepsilon)t} \frac{\sum_{j=0}^{\min(x_1 x_2)} \varepsilon(t)^j \ (\lambda_i - \varepsilon)t^{x_1 - j} \ (\mu - \varepsilon)t^{x_2 - j}}{j!\,(x_1 - j)!\,(x_2 - j)!}$$

$x_1, x_2 = 0,1,2,\ldots\ldots, \lambda_i, \mu < 0,\ i = 0,1;$

with parameters λ_0, λ_1, μ_n and ε as mean faster rate of primary arrivals, mean slower rate of primary arrivals, mean service rate and mean dependence rate (covariance between the primary arrival and service processes) respectively.

At time t, let N(t) be the number of sources of repeated calls and C(t) be the number of busy servers. The system state at time t can be described by means of a bivariate process {C(t),N(t)},t $\geq$ 0, where C(t)=1 or 0 according as the server is busy or idle, the process will be called CN process. If the service time is exponential, then C(t),N(t) is Markovian.

The process N(t),C(t) ;t $\geq$ 0 forms a Markov chain with state space (n,c)|n = {0,1,2,.....r-1,r,r+1,....R-1,R,R+1....K} ,c={ 0,1}. Let C and N be the numbers of customers in the service facility and in the orbit, respectively, in steady state. The state probabilities at time t are defined as follows

III. Steady State Equation

Let $P_{0,n,0}$ denote the steady state probability that there are n customers in the queue when the system is in the faster rate of primary arrivals and the server is idle.

Let $P_{0,n,1}$ denote the steady state probability that there are n customers in the queue when the system is in the slower rate of primary arrivals and the server is idle.

Let $P_{1,n,0}$ denote the steady state probability that there are n customers in the queue when the system is in the faster rate of primary arrivals and the server is busy.

Let $P_{1,n,1}$ denote the steady state probability that there are n customers in the queue when the system is in the slower rate of primary arrivals and the server is busy.

We observe that only $\$P_{0,n,0}$ and $\$P_{1,n,0}$ exists when n=0,1,2, …….., r-1,r; $P_{0,n,0}$, $P_{1,n,0}$, $P_{0,n,1}$ and $P_{1,n,1}$ exist when n=r+1,r+2,……, R-2,R-1; $P_{0,n,1}$ and $P_{1,n,1}$ exists when n=R,R+1,….. , K. Further $P_{0,n,0} = P_{1,n,0} = P_{0,n,1} = P_{1,n,1} = 0$ if n>K.

The steady state equations are

$$-N(\lambda_0 - \varepsilon)P_{0,0,0} + (\mu - \varepsilon)P_{1,0,0} = 0 \quad (1)$$

$$-[(N-1)H_1(\lambda_0 - \varepsilon) + (\mu - \varepsilon)]P_{1,0,0} + N(\lambda_0 - \varepsilon)P_{0,0,0} + \theta P_{0,1,0} + \theta(1 - H_2) P_{1,1,0} = 0 \quad (2)$$

$$-[(N-n)(\lambda_0 - \varepsilon) + n\theta]P_{0,n,0} + (\mu - \varepsilon)P_{1,n,0} = 0 \quad (3)$$

$$-[(N-n-1)H_1(\lambda_0 - \varepsilon) + (\mu - \varepsilon) + n\theta(1 - H_2) + (n-1)\alpha p]\, P_{1,n,0} + [(N-n)(\lambda_0 - \varepsilon)]\, P_{0,n,0} + [(N-n)H_1(\lambda_0 - \varepsilon)]\, P_{1,n-1,0} + (n+1)\theta\, P_{0,n+1,0} + [(n+1)\theta(1-H_2) + n\alpha p]\, P_{1,n+1,0} = 0,$$

$$n=1,2,3,\ldots\ldots,r-1 \quad (4)$$

$$-[(N-r)(\lambda_0 - \varepsilon) + r\theta]P_{0,r,0} + (\mu - \varepsilon)P_{1,r,0} = 0 \quad (5)$$

$$-[(N-r-1)H_1(\lambda_0 - \varepsilon) + (\mu - \varepsilon) + r\theta(1 - H_2) + (r-1)\alpha p]\, P_{1,r,0} + [(N-r)(\lambda_0 - \varepsilon)]\, P_{0,r,0} + [(N-r)H_1(\lambda_0 - \varepsilon)]\, P_{1,r-1,0} + (r+1)\theta\, P_{0,r+1,0} + [(r+1)\theta(1-H_2) + r\alpha p]\, P_{1,r+1,0} + (r+1)\theta\, P_{0,r+1,1} + [(r+1)\theta(1-H_2) + r\alpha p]\, P_{1,r+1,1} = 0 \quad (6)$$

$$-[(N-n)(\lambda_0 - \varepsilon) + n\theta]P_{0,n,0} + (\mu - \varepsilon)P_{1,n,0} = 0 \quad (7)$$

$$-[(N-n-1)H_1(\lambda_0 - \varepsilon) + (\mu - \varepsilon) + n\theta(1 - H_2) + (n-1)\alpha p]\, P_{1,n,0} + [(N-n)(\lambda_0 - \varepsilon)]\, P_{0,n,0} + [(N-n)H_1(\lambda_0 - \varepsilon)]\, P_{1,n-1,0} + (n+1)\theta\, P_{0,n+1,0} + [(n+1)\theta(1-H_2) + n\alpha p]\, P_{1,n+1,0} = 0,$$

$$n = r+1, r+2, \ldots\ldots, R-2 \quad (8)$$

$$-[(N-R+1)(\lambda_0 - \varepsilon) + (R-1)\theta]P_{0,R-1,0} + (\mu - \varepsilon)P_{1,R-1,0} = 0 \quad (9)$$

$$-[(N-R)H_1(\lambda_0 - \varepsilon) + (\mu - \varepsilon) + (R-1)\theta(1 - H_2)]\, P_{1,R-1,0} + [(N-R+1)(\lambda_0 - \varepsilon)]\, P_{0,R-1,0} + [(N-R+1)H_1(\lambda_0 - \varepsilon)]\, P_{1,R-2,0} = 0 \quad (10)$$

$$-[(N-r-1)(\lambda_1 - \varepsilon) + (r+1)\theta]P_{0,r+1,1} + (\mu - \varepsilon)P_{1,r+1,1} = 0 \quad (11)$$

$$-[(N-r-2)H_1(\lambda_1 - \varepsilon) + (\mu - \varepsilon) + (r+1)\theta(1 - H_2) + (r\alpha p)]\, P_{1,r+1,1} + [(N-r-1)(\lambda_1 - \varepsilon)]\, P_{0,r+1,1} + [(N-n)H_1(\lambda_1 - \varepsilon)]\, P_{0,r+1,1} + (r+2)\theta\, P_{0,r+2,1} + [(r+2)\theta(1-H_2) + (r+1)\alpha p]\, P_{1,r+2,1} = 0 \quad (12)$$

$$-[(N-n)(\lambda_1 - \varepsilon) + n\theta]P_{0,n,1} + (\mu - \varepsilon)P_{1,n,1} = 0 \quad (13)$$

$$-[(N-n-1)H_1(\lambda_1 - \varepsilon) + (\mu - \varepsilon) + n\theta(1 - H_2) + (n-1)\alpha p]\, P_{1,n,1} + [(N-n)(\lambda_1 - \varepsilon)]\, P_{0,n,1} + [(N-n)H_1(\lambda_1 - \varepsilon)]\, P_{1,n-1,1} + (n+1)\theta\, P_{0,n+1,1} + [(n+1)\theta(1-H_2) + n\alpha p]\, P_{1,n+1,1} = 0,$$

$$n = r+2, r+3, \ldots\ldots R-1 \quad (14)$$

$$-[(N-R)(\lambda_1 - \varepsilon) + R\theta]P_{0,R,1} + (\mu - \varepsilon)P_{1,R,1} = 0 \quad (15)$$

$$-[(N-R-1)H_1(\lambda_1 - \varepsilon) + (\mu - \varepsilon) + R\theta(1 - H_2) + (R-1)\alpha p]\, P_{1,R,1} + [(N-R)(\lambda_1 - \varepsilon)]\, P_{0,R,1} + [(N-R)H_1(\lambda_1 - \varepsilon)]\, P_{1,R-1,1} + [(N-R)H_1(\lambda_0 - \varepsilon)]\, P_{1,R-1,0} + (R+1)\theta\, P_{0,R+1,1} + [(R+1)\theta(1-H_2) + R\alpha p]\, P_{1,R+1,1} = 0 \quad (16)$$

$$-[(N-n)(\lambda_1 - \varepsilon) + n\theta]P_{0,n,1} + (\mu - \varepsilon)P_{1,n,1} = 0 \quad (17)$$

$$-[(N-n-1)H_1(\lambda_1 - \varepsilon) + (\mu - \varepsilon) + n\theta(1 - H_2 + (n-1)\alpha p)]\, P_{1,n,1} + [(N-n)(\lambda_1 - \varepsilon)]\, P_{0,n,1} + [(N-n)H_1(\lambda_1 - \varepsilon)]\, P_{1,n-1,1} + (n+1)\theta\, P_{0,n+1,1} + [(n+1)\theta(1-H_2) + n\alpha p]\, P_{1,n+1,1} = 0,$$

n = R+1, R+2, ………K-1 (18)

-[(N-K)$(\lambda_1 - \varepsilon)$+K$\theta$]$P_{0,K,1}$ + $(\mu - \varepsilon)P_{1,K,1}$=0 (19)

-$(\mu - \varepsilon)$ $P_{1,K,1}$ + [(N-K)$H_1(\lambda_1 - \varepsilon)$] $P_{1,K-1,1}$

[(N-K)$(\lambda_1 - \varepsilon)$] $P_{0,K,1}$ = 0 (20)

Write $\gamma = [H_1(\lambda_0 - \varepsilon)]$ and $\delta = [H_1(\lambda_1 - \varepsilon)]$

From (1) to (4) we get,

$$P_{0,n,0} = \frac{(N-1)_n \gamma^n \prod_{i=0}^{n-1}[(N-i)(\lambda_0-\varepsilon)+i\theta]}{\prod_{i=1}^{n} i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(N-i)(\lambda_0-\varepsilon)+i\theta]} P_{0,0,0}$$

1≤n≤r (21)

$$P_{1,n,0} = \frac{(N-1)_n \gamma^n}{\mu-\varepsilon} \prod_{i=1}^{n} \frac{[(N-i)(\lambda_0-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(N-i)(\lambda_0-\varepsilon)+i\theta]} P_{0,0,0}$$

(22)

From (5) to (8) we get,

$$P_{0,n,0} = \frac{(N-1)_n \gamma^n \prod_{i=0}^{n-1}[(N-i)(\lambda_0-\varepsilon)+i\theta]}{\prod_{i=1}^{n} i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(N-i)(\lambda_0-\varepsilon)+i\theta]} P_{0,0,0}$$
$$-\Bigg\{\frac{A_1}{A_2}\Bigg(\Bigg[\sum_{m-r}^{n-2}(N-m-2)\gamma^{n-1-m} \prod_{i=m+1}^{n} \frac{[(N-i)(\lambda_0-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(N-i)(\lambda_0-\varepsilon)+i\theta]}\Bigg] + 1\Bigg)\Bigg\} P_{0,r+1,1}$$

(23)

$$P_{1,n,0} = \frac{(N-1)_n \gamma^n}{\mu-\varepsilon} \prod_{i=1}^{n} \frac{[(N-i)(\lambda_0-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(N-i)(\lambda_0-\varepsilon)+i\theta]} P_{0,0,0}$$
$$-\Bigg\{\frac{A_1}{A_2(\mu-\varepsilon)}\Bigg(\Bigg[\sum_{m-r}^{n-1}(N-m-2)\gamma^{n-1-m} \prod_{i=m+1}^{n} \frac{[(N-i)(\lambda_0-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(N-i)(\lambda_0-\varepsilon)+i\theta]}\Bigg]\Bigg)\Bigg\} P_{0,r+1,1}$$

n = r+1,r+2,……,R-1 (24)

where

$A_1 = (r+1)\theta(\mu-\varepsilon)+[(r+1)\theta(1-H_2)+r\alpha p]$
$[(N-r-1)(\lambda_1-\varepsilon)+(r+1)\theta]$

$A_2 = n\theta(\mu-\varepsilon)+[n\theta(1-H_2)+(n-1)\alpha p]$
$[(N-n)(\lambda_0-\varepsilon)+n\theta]$

From (9) to (10) we get,

$P_{0,r+1,1} = \frac{A_3}{A_4} p_{0,0,0}$

Where

$$A_3 = \frac{(N-1)_R \gamma^R \prod_{i=0}^{R-1}[(N-i)(\lambda_0-\varepsilon)+i\theta]}{\prod_{i=1}^{R-1} i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(N-i)(\lambda_0-\varepsilon)+i\theta]} P_{0,0,0}$$

$$A_4 = -\Bigg\{A_1\Bigg(\Bigg[\sum_{m-r}^{R-2}(N-m-2)\gamma^{n-1-m} \prod_{i=m+1}^{R-1} \frac{[(N-i)(\lambda_0-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(N-i)(\lambda_0-\varepsilon)+i\theta]}\Bigg]\Bigg)\Bigg\} P_{0,r+1,1}$$

(25)

From (11) to (14), we recursively derive,

$$P_{0,n,1} = \Bigg\{\frac{A_1}{A_5}\Bigg(\Bigg[\sum_{m-r}^{n-2}(N-m-2)\delta^{n-1-m} \prod_{i=m+1}^{n-1} \frac{[(N-i)(\lambda_1-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(N-i)(\lambda_0-\varepsilon)+i\theta]}\Bigg] + 1\Bigg)\Bigg\} P_{0,r+1,1}$$

(26)

$$P_{1,n,1} = \Bigg\{\frac{A_1}{\mu-\varepsilon}\Bigg(\Bigg[\sum_{m-r}^{n-1}(N-m-2)\delta^{n-1-m} \prod_{i=m+1}^{n} \frac{[(N-i)(\lambda_1-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(N-i)(\lambda_1-\varepsilon)+i\theta]}\Bigg]\Bigg)\Bigg\} P_{0,r+1,1}$$

n= r+1,r+2,……R-1,R (27)

where

$A_5 = n\theta(\mu-\varepsilon)+[n\theta(1-H_2)+(n-1)\alpha p]$
$[(N-n)(\lambda_1-\varepsilon)+n\theta]$

A_1 is given by (23) and $P_{0,r+1,1}$ is given by (25)

From (15) to (20) we recursively derive,

$$P_{0,n,1} = \Bigg\{\frac{A_1}{A_5}\Bigg(\Bigg[\sum_{m-r}^{R-2}(N-m-2)\delta^{n-1-m} \prod_{i=m+1}^{n-1} \frac{[(N-i)(\lambda_1-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(N-i)(\lambda_0-\varepsilon)+i\theta]}\Bigg]\Bigg)\Bigg\} P_{0,r+1,1}$$

(28)

$$P_{1,n,1} = \Bigg\{\frac{A_1}{\mu-\varepsilon}\Bigg(\Bigg[\sum_{m-r}^{R-1}(N-m-2)\delta^{n-1-m} \prod_{i=m+1}^{n} \frac{[(N-i)(\lambda_1-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-H_2)+(i-1)\alpha p][(N-i)(\lambda_1-\varepsilon)+i\theta]}\Bigg]\Bigg)\Bigg\} P_{0,r+1,1}$$

n=R+1, R+2,…K-1,K (29)

where A_1, A_5 and $P_{0,r+1,1}$ are given by (23), (25), (26).

Thus from (21) to (29), we find that all the steady state probabilities are expressed in terms of $P_{0,0,0}$.

IV. Characteristics of The Model

The following system characteristics are considered and their analytical results are derived in this system.

- The probability P(0)that the system is in faster rate of primary arrivals with the server idle and busy.
- The probability P(1) that the system is in slower rate of primary arrivals with the server idle and busy.
- The probability $P_{0,0,0}$ that the system is empty.
- The expected number of customers in the system Ls_0, when the system is in faster rate of primary arrivals with the server idle and busy.
- The expected number of customers in the system Ls_1, when the system is in slower rate of primary arrivals with the server idle and busy.

The probability that the system is in faster rate of primary arrivals is

$$P(0) = \left[\sum_{n=0}^{K} P_{0,n,0} + \sum_{n=0}^{K} P_{1,n,0}\right]$$

$$= \begin{bmatrix}\sum_{n=0}^{r} P_{0,n,0} + \sum_{n=r+1}^{R-1} P_{0,n,0} \\ +\sum_{n=r+1}^{R-1} P_{0,n,0}\end{bmatrix} + \begin{bmatrix}\sum_{n=0}^{r} P_{1,n,0} + \sum_{n=r+1}^{R-1} P_{1,n,0} \\ +\sum_{n=R}^{K} P_{1,n,0}\end{bmatrix}$$

Since $P_{0,n,0}$ and $P_{1,n,0}$ exist only when n= 0,1,2,……, r-1,r,r+1,r+2,…..R-2,R-1, we get

$$P(0) = \left[\sum_{n=0}^{r} P_{0,n,0} + \sum_{n=r+1}^{R-1} P_{0,n,0}\right] + \left[\sum_{n=0}^{r} P_{1,n,0} + \sum_{n=r+1}^{R-1} P_{1,n,0}\right] \quad (30)$$

The probability that the system is in slower rate of primary arrivals is,

$$P(1) = \left[\sum_{n=0}^{K} P_{0,n,1} + \sum_{n=0}^{K} P_{1,n,1}\right]$$

$$= \begin{bmatrix}\sum_{n=0}^{r} P_{0,n,1} + \sum_{n=r+1}^{R-1} P_{0,n,1} \\ +\sum_{n=r+1}^{R-1} P_{0,n,1}\end{bmatrix} + \begin{bmatrix}\sum_{n=0}^{r} P_{1,n,1} + \sum_{n=r+1}^{R-1} P_{1,n,1} \\ +\sum_{n=R}^{K} P_{1,n,1}\end{bmatrix}$$

Since $P_{0,n,1}$ and $P_{1,n,1}$ exist only when n= r+1,r+2,….. R-2,R-1…..K, we get

$$P(1) = \left[\sum_{n=r+1}^{R} P_{0,n,1} + \sum_{n=R+1}^{K} P_{0,n,1}\right] + \left[\sum_{n=r+1}^{R} P_{1,n,1} + \sum_{n=R+1}^{K} P_{1,n,1}\right] \quad (31)$$

The probability $P_{0,0,0}$ that the system is empty can be calculated from the normalizing condition P(0) + P(1) =1. $P_{0,0,0}$ is calculated from (30) and (31) .

Let L_s denote the average number of customers in the system, then we have

$$L_s = L_{so} + L_{s1} \quad (32)$$

$$L_{s_0} = \left[\sum_{n=0}^{r} nP_{0,n,0} + \sum_{n=r+1}^{R-1} nP_{0,n,0}\right] + \left[\sum_{n=0}^{r}(n+1)P_{1,n,0} + \sum_{n=r+1}^{R-1}(n+1)P_{1,n,0}\right] \quad (33)$$

and

$$L_{s_1} = \left[\sum_{n=r+1}^{R} nP_{0,n,1} + \sum_{n=R+1}^{K} nP_{0,n,1}\right] + \left[\sum_{n=r+1}^{R}(n+1)P_{1,n,1} + \sum_{n=R+1}^{K}(n+1)P_{1,n,1}\right] \quad (34)$$

From (21) to (29) , (33) and (34) , we can calculate the value of L_s. The expected waiting time of the customers in the orbit is calculated as $W_s = \frac{L_s}{\bar{\lambda}}$, Where $\bar{\lambda} = \lambda_0$ P(0) + λ_1P(1). W_s is calculated from (30) to (32).

This model includes the following models as particular cases. For example, when $H_1 = 1$, $H_2 = 1$, p=0, $\alpha = 0$ we get the M/M/1/K/N interdependent retrial queueing model with controllable arrival rates. When $H_1 = 1$, $H_2 = 1$, $\alpha = 0$, p=0 and $\theta \to \infty$, we get the standard M/M/1/K/N interdependent queueing model with controllable arrival rates. When λ_0 tends to λ_1, p=1, $\epsilon = 0$ and infinite source this model reduces to M/M/1/K retrial queueing model with balking and reneging customers. When λ_0 tends to λ_1, $\epsilon = 0$, $H_1 = 1$ and $H_2 = 1$, $\theta \to \infty$, infinite source this model reduces to M/M/1/N queue with retention of reneged customers. When λ_0 tends to λ_1, $\epsilon = 0$, $H_1=1$, $H_2=1$, p=0, $\alpha = 0$ and $\theta \to \infty$, this model reduces to standard M/M/1/K/N queueing model.

V. Numerical Illustrations

For various values λ_0, λ_1, μ, ϵ, θ, α , p, N while r, R, K, H_1, H_2 are fixed values, computed and tabulated the values of $P_{0,0,0}$,P(0), P(1), L_s and W_s.

TABLE 1

r=3,R=6,K=8,H_1=0.8, H_2=0.2								
λ_0	λ_1	μ	θ	ϵ	α	p=1-q	N	$P_{0,0,0}$
3	2	4	2	0.5	0.1	0.4	10	3.4670155×10^{-4}
4	2	4	2	0.5	0.1	0.4	10	5.2238772×10^{-5}
4	3	4	2	0.5	0.1	0.4	10	4.0896436×10^{-5}
4	3	4	3	0.5	0.1	0.4	10	1.4703411×10^{-4}
4	3	4	3	0.5	0.1	0.4	11	9.4184131×10^{-5}
4	3	5	3	0.5	0.1	0.4	10	3.2178762×10^{-4}
3	2	4	2	1	0.1	0.4	10	1.5542188×10^{-4}
4	3	4	3	0.5	0.1	1	10	2.7459791×10^{-4}
3	2	4	2	0.5	0	0	10	2.6926279×10^{-4}

TABLE 2

P(0)	P(1)	L_s	W_s
0.298036875	0.701963125	5.076481742	2.209051472
0.049843111	0.95015689	5.557583062	2.646863617
0.038787671	0.961212328	6.129769643	2.01717603
0.121727476	0.878272524	3.109381261	0.99604507
0.206467401	0.793532599	5.273733544	1.644717655
0.430291789	0.569708211	2.689848833	0.78414578
0.049264609	0.950735391	6.058771001	2.956558648
0.316152413	0.683847587	3.086583413	0.969901382
0.264762025	0.735237975	6.205176895	2.739880319

VI. CONCLUSION

It is observed from the tables 1 and 2 that when λ_0 increases keeping the other parameters fixed, $P_{0,0,0}$ and P(0) decrease but P(1), Ls and Ws increase. When λ_1 increases keeping the other parameters fixed, $P_{0,0,0}$ and P(0) decrease but P(1), Ls and Ws increase. When θ increases keeping the other parameters fixed, $P_{0,0,0}$ and P(0) increase but P(1), Ls andWs decrease. When θ increases keeping the other parameters fixed, $P_{0,0,0}$ and P(0) increase but P(1), Ls and Ws decrease. When μ increases keeping the other parameters fixed, $P_{0,0,0}$ and P(0) increase but P(1), Ls and Ws decrease. When N increases keeping the other parameters fixed, $P_{0,0,0}$ and P(0) decrease but P(1), Ls and Ws increase. When p=1 keeping the other parameters fixed, $P_{0,0,0}$ and P(0) increase but P(1), Ls and Ws decrease. When $\alpha = 0$, p = 0 keeping the other parameters fixed, $P_{0,0,0}$ and P(0) decrease but P(1), Ls and Ws increase.

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