

Base stock policy inventory system with Multiple vacations and Negative customers

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Abstract- In this paper, we consider a continuous review base stock policy inventory system with multiple vacations and negative customers. The maximum storage capacity is S . The customers arrive according to a Poisson process with finite waiting hall. The customers are of two types : ordinary and negative. An ordinary customer, on arrival, joins the queue and the negative customer does not join the queue and takes away any one of the waiting customers. When the waiting hall is full, arriving primary customer is considered to be lost. The service time and lead time are assumed to have independent exponential distribution. When the inventory becomes empty, the server takes a vacation and the vacation duration is exponentially distributed. We obtained the joint probability distribution of the number of customers in the waiting hall, the inventory level and the server status for the steady state case. Some system performance measures are derived. The long-run total expected cost rate is calculated and numerical study is presented.

Keywords- Continuous review inventory system, Positive leadtime, Base Stock Policy, Multiple vacations, Negative Customers.

AMS Subject Classification: 90B05, 60J27.

I. INTRODUCTION

Analysis of continuous review perishable inventory systems with positive leadtimes under $(S-1, S)$ policy have been carried out by Schmidt and Nahmias [20], Pal [19], and Kalpakam and Sapna [14, 15]. In all these models, whenever the inventory level drops by one unit, either due to a demand or a failure, an order for one item is placed. Kalpakam and Arivarignan [13] discussed with an $(S-1, S)$ system with renewal demands for non-perishable items. Kalpakam and Shanthi [16] have considered modified base stock policy and random supply quantity. Recently, Gomathi et al. [11] considered a

two commodity inventory system for base-stock policy with service facility. They have assumed Poisson arrivals and the life time of each item and service time are assumed to have independent exponential distribution.

Berman et al. [2] have dealt an inventory management system at a service facility which uses one item of inventory for each service provided. Berman and Kim [3] considered a queueing - inventory system with Poisson arrivals, exponential service times and zero lead times. Berman and Sapna [4] studied the concept of queueing - inventory system with service facility. Krishnamoorthy and Anbazhagan [17] analyzed a perishable queueing inventory system with N policy, Poisson arrivals, exponential distributed lead times and service times. The joint probability distributions of the number of customers in the system and the inventory level were obtained in the steady state case. Jeganathan et al. [12] studied a retrial inventory system with non-preemptive priority service.

The concept of negative customer is increasingly considered in queueing systems. The customers who arrive at the service station are classified as ordinary (positive or regular) and negative customers. The arrival of ordinary customers to the service station increases the queue length by one and the arrival of negative customer to the service station causes one ordinary customer to be removed, if any is present. Since the work analysed by Gelenbe [10], the research on queueing systems with negative arrivals has been greatly motivated by some practical applications in computers, neural networks and communication networks etc. For comprehensive literature on queueing networks with negative arrivals, one may refer to Choa et al. [5], and Gelenbe and Pujolle [9]. A recent review can be found in Artalejo [1].

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Daniel and Ramanarayanan [6] have first introduced the server vacation in inventory with two servers. Also they have studied an inventory system in which the server takes rest when the level of the inventory is zero in [7]. They assumed that the demands that occurred during stock-out period are assumed to be lost. Narayanan et al. [18] studied on an (s, S) inventory policy with service time, vacation to server and correlated lead time. Sivakumar [21] has considered a retrial inventory system with multiple server vacation. He has assumed Poisson arrival and exponential service time. Further he assumed that the server takes a vacation of exponential length each time when the inventory level becomes zero.

In this paper we have considered a $(S-1, S)$ policy stochastic inventory system under continuous review at a service facility with a finite waiting hall for customers. The customers arriving to the service station are classified as ordinary and negative customers. The server takes a vacation of exponential length each time when the inventory level becomes empty. When the vacation ends he finds the inventory level is still zero, the server takes another vacation; otherwise, he terminates his vacation, and he is ready to serve any arriving demands. The joint probability distribution of the number of customers in the waiting hall, the inventory level and the server status is obtained for the steady state case. Various system performance measures in the steady state are derived and the long-run total expected cost rate is calculated.

The remainder of this paper is organized as follows. In Section 2, we present the mathematical model and the notations. Analysis of the model and the steady state solution are given in Section 3. In Section 4, we derive various measures of system performance in steady state. In Section 5, The total expected cost rate is calculated and numerical study is presented. The last section is meant for conclusion.

II. MODEL DESCRIPTION

We consider a single server continuous review stochastic inventory system with multiple vacations and negative customers. The maximum inventory level is denoted by S . The primary customers arrive at the system one by one in according to a Poisson stream with arrival rate $\lambda (> 0)$. Waiting hall space is limited to accommodate a maximum number M , which includes the customer one who is receiving service. The probability that a customer is an

ordinary is p and a negative is $q (= 1 - p)$. We have assumed that the negative customer removes any one of the ordinary waiting customers from the system including the one at the service point. The service time for each customer follows an exponential distribution with parameter μ . An any arriving primary customer who finds the waiting hall is full is considered to be lost. A one-to-one ordering policy is adopted. According to this policy, orders are placed for one unit as and when the inventory level drops due to a demand. The lead time is exponentially distributed with the rate β . The server leaves for a vacation as soon as the inventory becomes empty. After the vacation, if the inventory level is positive, he begins to serve the customers right away otherwise he takes another vacation. The vacation duration is exponentially distributed with rate θ . We assume that the inter-demand times between primary customers, the lead times, service times and the server vacation times are mutually independent random variables.

Notations

$[A]_{ij}$: The element / submatrix at (i, j) th position of A .

$\mathbf{0}$: Zero matrix.

I : Identity matrix.

e : A column vector of 1's of appropriate dimension.

δ_{ij} : $\begin{cases} 1, & \text{if } i = j, \\ 0, & \text{otherwise.} \end{cases}$

$\bar{\delta}_{ij}$: $1 - \delta_{ij}$

$Y(t)$: $\begin{cases} 0, & \text{if server is on vacation at time } t. \\ 1, & \text{if server is not on vacation at time } t. \end{cases}$

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III. ANALYSIS

Let $X(t)$, $L(t)$ and $Y(t)$ denote the number of customers in the waiting hall, the inventory level of the commodity and the server status at time t . From the assumptions made on the input and output processes, it can be shown that the triplet $\{(X(t), L(t), Y(t)), t \geq 0\}$ is a continuous time Markov chain with state space given by E .

$$E = \{(i, 0, 0) : i = 0, 1, 2, \dots, M\} \cup \{(i, k, m) : i = 0, 1, 2, \dots, M, k = 1, 2, \dots, S, m = 1, 0\}$$

To determine the infinitesimal generator

$$P = (h((i, k, m), (j, l, n))), \quad (i, k, m), (j, l, n) \in E$$

of this process we use the following arguments :

Transitions due to the arrival of an ordinary customers:

• $(i, k, m) \rightarrow (i+1, k, m)$: the rate is $p\lambda$, for $0 \leq i \leq M-1, 1 \leq k \leq S, m = 1, 0$.

• $(i, 0, 0) \rightarrow (i+1, 0, 0)$: the rate is $p\lambda$, for $0 \leq i \leq M-1$.

Transitions due to the arrival of a negative customers:

• $(i, k, m) \rightarrow (i-1, k, m)$: the rate is $q\lambda$, for $1 \leq i \leq M, 1 \leq k \leq S, m = 1, 0$.

• $(i, 0, 0) \rightarrow (i-1, 0, 0)$: the rate is $q\lambda$, for $1 \leq i \leq M$.

Transitions due to service completion in the system:

• $(i, k, 1) \rightarrow (i-1, k-1, 1)$: the rate is μ , for $1 \leq i \leq M, 2 \leq k \leq S$.

• $(i, 1, 1) \rightarrow (i-1, 0, 0)$: the rate is μ , for $1 \leq i \leq M$.

Transitions due to replenishments:

• $(i, k, m) \rightarrow (i, k+1, m)$: the rate is $(S-k)\beta$, for $0 \leq i \leq M, 1 \leq k \leq S, m = 1, 0$.

• $(i, 0, 0) \rightarrow (i, 1, 0)$: the rate is $S\beta$, for $0 \leq i \leq M$.

Transitions due to vacation completion:

• $(i, k, 0) \rightarrow (i, k, 1)$: the rate is θ , for

$$0 \leq i \leq M, 1 \leq k \leq S.$$

We observe that no transition other than the above is possible.

Denoting

$$q = ((q, 0, 0), (q, 1, 0), (q, 1, 1), (q, 2, 0), (q, 2, 1), \dots, (q, S, 0), (q, S, 1))$$

for $q = 0, 1, \dots, M$. By ordering states lexicographically, the infinitesimal generator A can be conveniently expressed in a block partitioned matrix with entries

$$[A]_{ij} = \begin{cases} A_2, & j = i, & i = M \\ A_1, & j = i, & i = 1, 2, \dots, M-1 \\ A_0, & j = i, & i = 0 \\ B, & j = i+1, & i = 0, 1, 2, \dots, M-1 \\ C, & j = i-1, & i = 1, 2, \dots, M \\ 0, & \text{otherwise} \end{cases}$$

where

$$[A_0]_{ij} = \begin{cases} G_{S-i}, & j = i, & i = S, S-1, \dots, 1 \\ G_{01}, & j = 1, & i = 0 \\ G_{00}, & j = 0, & i = 0 \\ 0, & \text{otherwise} \end{cases}$$

$$[G_{S-i}]_{mn} = \begin{cases} \theta, & n = 1, & m = 0 \\ (S-i)\beta, & n = i, & m = 1, 0 \\ -((S-i)\beta + p\lambda), & n = 1, & m = 1 \\ -((S-i)\beta + p\lambda + \theta), & n = 0, & m = 0 \\ 0, & \text{otherwise} \end{cases}$$

For $i = S, S-1, \dots, 1$

$$G_{01} = 0 \begin{pmatrix} \mathbf{1} & 0 \\ 0 & S\beta \end{pmatrix}, G_{00} = 0 \begin{pmatrix} 0 \\ -(S\beta + p\lambda) \end{pmatrix},$$

$$[A_1]_{ij} = \begin{cases} F_{S-i}, & j = i, & i = S, S-1, \dots, 1 \\ G_{01}, & j = 1, & i = 0 \\ F_{00}, & j = 0, & i = 0 \\ 0, & \text{otherwise} \end{cases}$$

$$[P_{S-i}]_{nm} = \begin{cases} \theta, & n = 1, & m = 0 \\ (S-i)\beta, & n = i, & m = 1, 0 \\ -((S-i)\beta + q\lambda + \mu), & n = 1, & m = 1 \\ -((S-i)\beta + q\lambda + \theta), & n = 0, & m = 0 \\ 0, & \text{otherwise} \end{cases}$$

For $i = S, S-1, \dots, 1$

$$P_{00} = 0 \begin{pmatrix} 0 \\ -(S\beta + q\lambda) \end{pmatrix},$$

$$B = p\lambda I_{(2S+1) \times (2S+1)}$$

$$C = q\lambda I_{(2S+1) \times (2S+1)}$$

It may be noted that the matrices A_0, A_1, A_2, B and C are square matrices of order $(2S+1)$

3.1 Steady State Analysis

It can be seen from the structure of A that the homogeneous Markov process $\{(X(t), L(t), Y(t)), t \geq 0\}$ on the finite state space E is irreducible. Hence the limiting distribution

$$\pi^{(i,j,k)} = \lim_{t \rightarrow \infty} pr\{X(t) = i, L(t) = j, Y(t) = k \mid X(0), L(0), Y(0)\}$$

exists. Let

$$\Pi = (\Pi^{(0)}, \Pi^{(1)}, \Pi^{(2)}, \dots, \Pi^{(M)})$$

we partition the vector, $\Pi^{(i)}$ into as follows:

$$\Pi^{(i)} = (\Pi^{(i,0)}, \Pi^{(i,1)}, \Pi^{(i,2)}, \dots, \Pi^{(i,S)}),$$

$$i = 0, 1, 2, \dots, M$$

which is partitioned as follows:

$$\Pi^{(i,j)} = (\pi^{(i,0,0)})$$

For $i = S, S-1, \dots, 1$

$$F_{00} = 0 \begin{pmatrix} 0 \\ -(S\beta + \lambda) \end{pmatrix},$$

$$[A_2]_{ij} = \begin{cases} P_{S-i}, & j = i, & i = S, S-1, \dots, 1 \\ G_{01}, & j = 1, & i = 0 \\ P_{00}, & j = 0, & i = 0 \\ 0, & \text{otherwise} \end{cases}$$

$$\Pi^{(i,j)} = (\pi^{(i,j,0)}, \pi^{(i,j,1)})$$

for $i = 0, 1, 2, \dots, M$, $j = 1, 2, \dots, S$

Then the limiting probability, Π satisfies

$$\Pi A = 0, \quad \Pi e = 1$$

From the structure of A , it is a finite QBD matrix, therefore its steady state vector Π can be computed by using the following algorithm described by Gaver et al. [8].

Algorithm :

1. Determine recursively the matrices

$$F_0 = A_0$$

$$F_i = A_1 + B(-F_{i-1}^{-1})C, \quad i = 1, 2, \dots, M-1,$$

$$F_M = A_2 + B(-F_{M-1}^{-1})C.$$

2. Compute recursively the vectors $\Pi^{(i)}$ using

$$\Pi^{(i)} = \Pi^{(i+1)}B(-F_i^{-1}), \quad i = 0, 1, 2, \dots, M-1$$

3. Solve the system of equations

$$\Pi^{(M)}F_M = 0 \text{ and } \sum_{i=0}^M \Pi^{(i)}e = 1.$$

From the system of equations $\Pi^{(M)}F_M = 0$, vector $\Pi^{(M)}$ could be determined uniquely, upto a multiplicative constant. This constant is decided by $\Pi^{(i)} = \Pi^{(i+1)}B(-F_i^{-1})$, $i = 0, 1, 2, \dots, M-1$ and $\sum_{i=0}^M \Pi^{(i)}e = 1$.

IV. SYSTEM PERFORMANCE MEASURES

In this section some performance measures of the system under consideration in the steady state are derived.

4.1 Expected inventory level

Let ρ_I denote the mean inventory level in the steady state. Then

$$\rho_I = \sum_{i=0}^M \sum_{j=1}^S j [\pi^{(i,j,1)} + \pi^{(i,j,0)}]$$

4.2 Expected reorder rate

Let ρ_R denote the expected reorder rate in

the steady state. Then

$$\rho_R = \sum_{i=1}^M \sum_{j=1}^S \mu [\pi^{(i,j,1)}]$$

4.3 Expected number of demands in the waiting hall

Let ρ_W denote the expected number of demands in the waiting hall in the steady state. Then

$$\rho_W = \sum_{i=1}^M \sum_{j=1}^S i [\pi^{(i,j,1)} + \pi^{(i,j,0)} + \pi^{(i,0,0)}]$$

4.4 Fraction of time server is on vacation

Let ρ_{FV} denote the server is on vacation in the steady state. Then

$$\rho_{FV} = \sum_{i=0}^M \sum_{j=0}^S [\pi^{(i,j,0)}]$$

4.5 Expected blocking rate

Let ρ_B denote the expected blocking rate in the steady state. Then

$$\rho_B = \sum_{j=1}^S p \lambda [\pi^{(M,j,1)} + \pi^{(M,j,0)} + \pi^{(M,0,0)}]$$

4.6 Mean rate of arrivals of negative customers

Let ρ_{Ng} denote mean rate of arrivals of negative customers in the steady state. Then

$$\rho_{Ng} = \sum_{i=1}^M \sum_{j=1}^S q \lambda [\pi^{(i,j,1)} + \pi^{(i,j,0)} + \pi^{(i,0,0)}]$$

V. COST ANALYSIS

The expected total cost per unit time (expected total cost rate) in the steady state for this model is defined to be

c_h : the inventory holding cost per unit item per unit time

c_s : the inventory setup cost per unit item per unit time

c_b : cost per blocking customer

c_w : Waiting cost of a customer in the waiting hall per unit time.

c_n : Cost of loss per unit time due to arrival of a

negative customer.

The long run total expected cost rate is given by

$$TC(S, M) = c_h \rho_I + c_s \rho_R + c_b \rho_B + c_w \rho_W + c_n \rho_{Ng}$$

Substituting ρ 's into the above equation, we obtain

$$TC(S, M) = c_h \sum_{i=0}^M \sum_{j=1}^S j [\pi^{(i,j,1)} + \pi^{(i,j,0)}] + c_s \sum_{i=1}^M \sum_{j=1}^S \mu [\pi^{(i,j,1)}] + c_b \sum_{j=1}^S p \lambda$$

$$[\pi^{(M,j,1)} + \pi^{(M,j,0)} + \pi^{(M,0,0)}] + c_w \sum_{i=1}^M \sum_{j=1}^S i [\pi^{(i,j,1)} + \pi^{(i,j,0)} + \pi^{(i,0,0)}] +$$

$$c_n \sum_{i=1}^M \sum_{j=1}^S q \lambda [\pi^{(i,j,1)} + \pi^{(i,j,0)} + \pi^{(i,0,0)}]$$

5.1 Numerical Examples

Since the total expected cost rate is obtained in a complex form, the convexity of the total expected cost rate cannot be studied by analytical methods. Hence, simple numerical search procedures are used to find the local optimal values for any two of the decision variable (S, M) by considering a small set of integer values for this variable. Table 1 presents the optimal value of the total expected cost rate for various combinations of the primary demand rate λ and the service rate μ . We have assumed constant values for other parameters and costs. Namely, $S = 5$, $M = 5$, $\theta = 0.05$, $\beta = 0.7$, $p = 0.7$, $q = 0.3$, $c_h = 0.98$, $c_s = 1.2$, $c_w = 2.09$, $c_n = 0.03$. The optimal value of the total expected cost rate is $TC^*(5, 5) = 12.659332$ for the values of $\lambda = 2.3$ and $\mu = 0.9$. The value that is shown bold is the least among the values in that column and the value that is shown underlined is the least in that row. Convexity of the total cost for various combinations of λ and μ is given in figure 1.

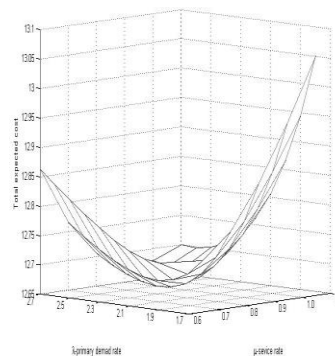


Figure 1: Convexity of the total cost for various combinations of λ and μ

Table 1: Total expected cost rate as a function of λ and μ

μ	0.6	0.7	0.8	0.9	1.0	1.1
λ						
1.7	12.733593	<u>12.729977</u>	12.772855	12.848242	12.943846	13.050069
1.8	12.728964	<u>12.703866</u>	12.724866	12.779530	12.856327	12.945879
1.9	12.732824	<u>12.690244</u>	12.693029	12.730161	12.790871	12.866053
2.0	12.742915	12.686194	12.673904	12.696304	12.743353	12.806243
2.1	12.757527	12.689439	<u>12.664740</u>	12.674827	12.710343	12.762782
2.2	12.775365	12.698203	12.663342	<u>12.663189</u>	12.689012	12.732606
2.3	12.795448	12.711102	12.667966	<u>12.659332</u>	12.677039	12.713172
2.4	12.817035	12.727060	12.677224	<u>12.661596</u>	12.672532	12.702388
2.5	12.839564	12.745239	12.690016	<u>12.668646</u>	12.673950	12.698534
2.6	12.862613	12.764987	12.705470	<u>12.679410</u>	12.680042	12.700207

VI. CONCLUSION

In this paper, we discussed continuous review inventory system with base stock policy. The customers are of two type : ordinary and negative. Various system performance measures are derived in the steady state. The results are illustrated with numerically. The model discussed here is useful in studying a multiple vacation for the base-stock policy which are slow moving items and the high holding cost.

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