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Stochastic models for a two grade manpower system having thresholds with two components

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Abstract— **In this paper an organization with two grades is considered in which exit of personnel takes place due to policy decisions announced. In order to avoid the crisis of the organization reaching a breakdown point, a suitable univariate policy of recruitment based on shock model approach is suggested which is used to enable the organization for planning its decision on recruitment. Three mathematical models are constructed and the expected and variance of time for recruitment are obtained when (i) the loss of manpower form a sequence of identically independent distributed random variables and (ii) the threshold for each grade has two components. The influence of the nodal parameters on the system characteristics are studied and relevant conclusions are presented.**

Keywords— **Man power planning; Univariate recruitment policy; Geometric process; correlated random variables; Mean and variance of the time to recruitment; Shock model; Hyper exponential distribution.**

I. INTRODUCTION

Exits of personnel is a common phenomenon in any marketing organization. This leads to reduction in the total strength of marketing personnel and will adversely affect the sales turnover of the organization, if the recruitment is not planned. In fact, frequent recruitment may also be expensive due to the cost of recruitment and training. As the loss of manpower is unpredictable, a suitable recruitment policy has to be designed to overcome this loss. In [1] and [2] several stochastic models for a manpower system with grades are discussed using Markovian and renewal theoretic approach. In [4] the authors have initiated the study on problem of time to recruitment for a single grade manpower system when the inter-decision times are independent and identically distributed random variables using shock model approach. In [8] the authors have studied the work in [4] when the breakdown threshold has a normal component and a component due to frequent breaks. In [5],[6],[7] the authors have obtained the mean and variance of time to recruitment for a two grade manpower system when the threshold for each grade has only the normal component. The present paper studies the results of [5],[6],[7] when the threshold for each grade has two components. This paper is organized as follows: In sections II, III and IV Models 1, 2 and 3 are described and analytical expressions for mean and variance of the time to recruitment are derived. The three models are differ from each other in the context of permitting or not permitting transfer of personnel between two grades and providing a better allowable loss of manpower in the organization. More specifically, in model-1, transfer of personnel between the two grades is not permitted, in Model-2this transfer is permitted. In Model-3 the thresholds for the loss of man-hours in the two grades are combined in order to provide a better allowable loss of man-hours in the organization compared to Models 1 and 2. In section V, the analytical results are numerically illustrated and relevant conclusions are given.

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II. MODEL DESCRIPTION AND ANALYSIS FOR MODEL-1

Consider an organization having two grades in which decisions are taken at random epochs in $[0,\infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of man-hours to the organization, if a person quits and it is linear and cumulative. Let X_i be an exponential random variable with mean 1/c,(c>0) denoting the loss of man-hours in the organization at the i^{th} decision epoch, i=1, 2,3…with probability density function g(.). Let S_n be the cumulative wastage in man-hour, in the first 'n' decisions. Let $U_i, i=1,2,3...$ be the time between i-1^{th} and i^{th} decisions. The best distribution when the inter-decision times have high or low intensity of attrition is the hyper exponential distribution. Let $U_i, i=1,2,3...k$ are independent and identically distributed hyper exponential random variables with distribution (density) function F(.)(f(.)), and high(low) attrition rate $\lambda_h(\lambda_l)$ and p(q) be the proportion of decisions having high (low) attrition rate. Let $F_k(t)$ $(f_k(t))$ be the distribution(probability density) function of $\sum_{i=1}^{k} U_i$. Let T be a continuous random variable denoting the time for recruitment in the organization with probability distribution function (density function) $L(.)(\ell(.))$. Let $l^*(.), g^*(.)$ *and* $f^*(.)$ be the Laplace transform of $\ell(.), g(.)$ *and* $f(.)$ respectively. Let Y be the breakdown threshold for the cumulative loss of manpower in the organization. For grade A(B), let Y_{A1} (Y_{B1}) be the normal exponential threshold for depletion of manpower with positive mean $\alpha_{A1}(\alpha_{B1})$ and Y_{A2} (Y_{B2}) be the exponential threshold of frequent breaks of existing workers with positive mean α_{A2} (α_{B2}). In this model, the breakdown threshold for the organization Y is taken as $\min(Y_A,Y_B)$. The loss of man-hours process and the inter-decision time process are statistically independent. The univariate recruitment policy employed in this paper is as follows: ***Recruitment is done as and when the total loss of man-hours in the organization exceeds Y.*** Let $V_k(t)$ be the probability that there are exactly k-decision epochs in (0,t]. Since the number of decisions made in (0,t] form a renewal process, we note that $V_k(t)= F_k(t) - F_{k+1}(t)$, where $F_0(t)=1$. Let E(T) and V(T) be the mean and variance of time for recruitment respectively.

Main results

By definition, $S_{N(t)}$ is the total loss of man-hours in the N(t) decisions taken in (0,t].

Therefore

$$P(T>t)=P\left(S_{N(t)}<Y\right) \tag{1}$$

By using laws of probability and on simplification we get

$$\begin{aligned}P(T>t) &= \gamma_1\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left(g^*(\alpha_{A1}+\alpha_{B1})\right)^k + \\ &\gamma_2\sum_{n=1}^{k}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{A2}+\alpha_{B2})\right]^{k-1} - \\ &\gamma_3\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{A1}+\alpha_{B2})\right]^{k-1} - \\ &\gamma_4\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{A2}+\alpha_{B1})\right]^{k-1}\end{aligned} \tag{2}$$

where $\gamma_1=\dfrac{\alpha_{A2}\alpha_{B2}}{(\alpha_{A2}-\alpha_{A1})(\alpha_{B2}-\alpha_{B1})}, \gamma_2=\dfrac{\alpha_{A1}\alpha_{B1}}{(\alpha_{A2}-\alpha_{A1})(\alpha_{B2}-\alpha_{B1})},$

$\gamma_3=\dfrac{\alpha_{A2}\alpha_{B1}}{(\alpha_{A2}-\alpha_{A1})(\alpha_{B2}-\alpha_{B1})}$ and $\gamma_4=\dfrac{\alpha_{A1}\alpha_{B2}}{(\alpha_{A2}-\alpha_{A1})(\alpha_{B2}-\alpha_{B1})}$

Since $L(t)=1-P(T>t)$

$$\begin{aligned}L(t) = &\gamma_1[1-g^*(\alpha_{A1}+\alpha_{B1})]\sum_{k=1}^{\infty}F_k(t)\left(g^*(\alpha_{A1}+\alpha_{B1})\right)^{k-1} + \\ &\gamma_2[1-g^*((\alpha_{A2}+\alpha_{B2}))]\sum_{k=1}^{\infty}F_k(t)\left[g^*(\alpha_{A2}+\alpha_{B2})\right]^{k-1} - \\ &\gamma_3[1-g^*(\alpha_{A1}+\alpha_{B2})]\sum_{k=1}^{\infty}F_k(t)\left[g^*(\alpha_{A1}+\alpha_{B2})\right]^{k-1} - \\ &\gamma_4[1-g^*(\alpha_{A2}+\alpha_{B1})]\sum_{k=1}^{\infty}F_k(t)\left[g^*(\alpha_{A2}+\alpha_{B1})\right]^{k-1}\end{aligned} \tag{3}$$

From(3) it is found that

$$\begin{aligned}\ell^*(s) = &\gamma_1\frac{\left[1-g^*(\alpha_{A1}+\alpha_{B1})\right]f^*(s)}{1-f^*(s)g^*(\alpha_{A1}+\alpha_{B1})}+\gamma_2\frac{\left[1-g^*(\alpha_{A2}+\alpha_{B2})\right]f^*(s)}{1-f^*(s)g^*(\alpha_{A2}+\alpha_{B2})} \\ &-\gamma_3\frac{\left[1-g^*(\alpha_{A1}+\alpha_{B2})\right]f^*(s)}{1-f^*(s)g^*(\alpha_{A1}+\alpha_{B2})}-\gamma_4\frac{\left[1-g^*(\alpha_{A2}+\alpha_{B1})\right]f^*(s)}{1-f^*(s)g^*(\alpha_{A2}+\alpha_{B1})}\end{aligned} \tag{4}$$

It is known that

$$E[T]=-\left.\frac{d(\ell^*(s))}{ds}\right|_{s=0}, E\left[T^2\right]=\left.\frac{d^2(\ell^*(s))}{ds^2}\right|_{s=0} \text{ and } V[T]=E\left[T^2\right]-\left(E[T]\right)^2 \tag{5}$$

From (4) and (5) it can be shown that

$$E[T]=(M_1)\left[\begin{array}{l}\dfrac{\gamma_1}{1-g^*(\alpha_{A1}+\alpha_{B1})}+\dfrac{\gamma_2}{1-g^*(\alpha_{A2}+\alpha_{B2})}- \\ \dfrac{\gamma_1}{1-g^*(\alpha_{A1}+\alpha_{B2})}-\dfrac{\gamma_1}{1-g^*(\alpha_{A2}+\alpha_{B1})}\end{array}\right] \tag{6}$$

and

$$\begin{aligned}E[T^2]=2(M_2)&\left[\begin{array}{l}\dfrac{\gamma_1}{\left(1-g^*(\alpha_{A1}+\alpha_{B1})\right)^2}+\dfrac{\gamma_2}{\left(1-g^*(\alpha_{A2}+\alpha_{B2})\right)^2}- \\ \dfrac{\gamma_1}{\left(1-g^*(\alpha_{A1}+\alpha_{B2})\right)^2}-\dfrac{\gamma_1}{\left(1-g^*(\alpha_{A2}+\alpha_{B1})\right)^2}\end{array}\right]- \\ 2(M_1)^2&\left[\begin{array}{l}\dfrac{\gamma_1 g^*(\alpha_{A1}+\alpha_{B1})}{\left(1-g^*(\alpha_{A1}+\alpha_{B1})\right)}+\dfrac{\gamma_2 g^*(\alpha_{A2}+\alpha_{B2})}{\left(1-g^*(\alpha_{A2}+\alpha_{B2})\right)}- \\ \dfrac{\gamma_1 g^*(\alpha_{A1}+\alpha_{B2})}{\left(1-g^*(\alpha_{A1}+\alpha_{B2})\right)}-\dfrac{\gamma_1 g^*(\alpha_{A2}+\alpha_{B1})}{\left(1-g^*(\alpha_{A2}+\alpha_{B1})\right)}\end{array}\right]\end{aligned} \tag{7}$$

where $M_1=\dfrac{p\lambda_l+q\lambda_h}{\lambda_h\lambda_l}$ $M_2=\dfrac{(p\lambda_l+q\lambda_h)^2}{{\lambda_h}^2{\lambda_l}^2}$ and $g^*(\tau)=\dfrac{c}{c+\tau}$

We shall obtain the E[T] and V[T] by considering different cases on U_i,i=1,2,3…

Note 1.

Assume that the inter-decision times U_i form a geometric process with parameter 'a'.

Since $\{U_i\}$ is a geometric process it is known that

$$f_k^*(s)=\prod_{n=1}^{k}f^*\left(s/a^{n-1}\right) \tag{8}$$

From (2),(5) and (8) we get

$$E[T]=a\,(M_1)\left[\begin{array}{l}\dfrac{\gamma_1}{a-g^*(\alpha_{A1}+\alpha_{B1})}+\dfrac{\gamma_2}{a-g^*(\alpha_{A2}+\alpha_{B2})}- \\ \dfrac{\gamma_3}{a-g^*(\alpha_{A1}+\alpha_{B2})}-\dfrac{\gamma_4}{a-g^*(\alpha_{A2}+\alpha_{B1})}\end{array}\right] \tag{9}$$

and

$$\begin{aligned}E[T^2]=\left[a^2M_2\right.&\left[\begin{array}{l}\dfrac{\gamma_1}{a^2-g^*(\alpha_{A1}+\alpha_{B1})}+\dfrac{\gamma_2}{a^2-g^*(\alpha_{A2}+\alpha_{B2})}- \\ \dfrac{\gamma_3}{a^2-g^*(\alpha_{A1}+\alpha_{B2})}-\dfrac{\gamma_4}{a^2-g^*(\alpha_{A2}+\alpha_{B1})}\end{array}\right]- \\ &(M_1)^2\left(\sum_{k=0}^{\infty}\left[\left(\sum_{i=1}^{k}\frac{1}{a^{i-1}}\right)^2-\left(\sum_{i=1}^{k+1}\frac{1}{a^{i-1}}\right)^2\right]\right)\times \\ &\left(\begin{array}{l}\gamma_1\left(g^*(\alpha_{A1}+\alpha_{B1})\right)^k+\gamma_2\left(g^*(\alpha_{A2}+\alpha_{B2})\right)^k- \\ \gamma_3\left(g^*(\alpha_{A1}+\alpha_{B2})\right)^k-\gamma_2\left(g^*(\alpha_{A2}+\alpha_{B1})\right)^k\end{array}\right)\end{aligned} \tag{10}$$

Note 2.

Suppose U_i are exchangeable and constantly correlated exponential random variables.

From Gurland [3] $F_k^*(s)$ is given by

$$F_k^*(s)=\frac{m^k}{1+\frac{kR(1-m)}{1-R}} \tag{11}$$

where R is the correlation between U_i and U_j , i # j ,
v = mean of U_i i=1,2,3… m=1/1+ds and d=v(1-R)

From (11) it is shown that

$$\frac{d}{ds}\left[F_k^*(s)\right]=-kv \tag{12}$$

$$\frac{d^2}{ds^2}\left[F_k^*(s)\right]=k\,v^2(1-R^2)+k^2\,v^2(1+R^2) \tag{13}$$

From (3),(5),(11),(12) and (13) it is shown that

$$E[T]=v\left[\begin{array}{l}\frac{\gamma_1}{1-g^*(\alpha_{A1}+\alpha_{B1})}+\frac{\gamma_2}{1-g^*(\alpha_{A2}+\alpha_{B2})}-\\ \frac{\gamma_3}{1-g^*(\alpha_{A1}+\alpha_{B2})}-\frac{\gamma_1}{1-g^*(\alpha_{A2}+\alpha_{B1})}\end{array}\right] \tag{14}$$

and

$$E[T^2]=2\times v^2\left[\begin{array}{l}\frac{\gamma_1(1+R^2)g^*(\alpha_{A1}+\alpha_{B1})}{(1-g^*(\alpha_{A1}+\alpha_{B1}))^2}+\frac{\gamma_2(1+R^2)g^*(\alpha_{A2}+\alpha_{B2})}{(1-g^*(\alpha_{A2}+\alpha_{B2}))^2}-\\ \frac{\gamma_3(1+R^2)g^*(\alpha_{A1}+\alpha_{B2})}{(1-g^*(\alpha_{A1}+\alpha_{B2}))^2}-\frac{\gamma_1(1+R^2)g^*(\alpha_{A2}+\alpha_{B1})}{(1-g^*(\alpha_{A2}+\alpha_{B1}))^2}\end{array}\right] \tag{15}$$

(15)

III. MODEL DESCRIPTION AND ANALYSIS FOR MODEL-2

For this model, $Y=\max(Y_A,Y_B)$. All the otherassumptions and notations are as in Model-I. In this model it can be shown that

$$\begin{aligned}P(T>t)&=\gamma_3\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{A1}+\alpha_{B2})\right]^k+\\ &\gamma_5\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{A1})\right]^k+\gamma_4\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{A2}+\alpha_{B1})\right]^k+\\ &\gamma_7\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{B1})\right]^k-\gamma_6\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{A2})\right]^k-\\ &\gamma_8\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{B2})\right]^k-\gamma_1\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left(g^*(\alpha_{A1}+\alpha_{B1})\right)^k-\\ &\gamma_2\sum_{n=1}^{k}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{A2}+\alpha_{B2})\right]^k\end{aligned} \tag{16}$$

where $\gamma_5=\frac{\alpha_{A2}}{(\alpha_{A2}-\alpha_{A1})},\gamma_6=\frac{\alpha_{A1}}{(\alpha_{A2}-\alpha_{A1})},\gamma_7=\frac{\alpha_{B2}}{(\alpha_{B2}-\alpha_{B1})}$

and $\gamma_8=\frac{\alpha_{B1}}{(\alpha_{B2}-\alpha_{B1})}$

Proceeding as in Model-I we get

$$\begin{aligned}\ell^*(s)=&\gamma_3\frac{\left[1-g^*(\alpha_{A1}+\alpha_{B2})\right]f^*(s)}{1-f^*(s)g^*(\alpha_{A1}+\alpha_{B2})}+\gamma_4\frac{\left[1-g^*(\alpha_{A2}+\alpha_{B1})\right]f^*(s)}{1-f^*(s)g^*(\alpha_{A2}+\alpha_{B1})}+\\ &\gamma_5\frac{\left[1-g^*(\alpha_{A1})\right]f^*(s)}{1-f^*(s)g^*(\alpha_{A1})}+\gamma_7\frac{\left[1-g^*(\alpha_{B1})\right]f^*(s)}{1-f^*(s)g^*(\alpha_{B1})}-\\ &\gamma_6\frac{\left[1-g^*(\alpha_{A2})\right]f^*(s)}{1-f^*(s)g^*(\alpha_{A2})}-\gamma_8\frac{\left[1-g^*(\alpha_{B2})\right]f^*(s)}{1-f^*(s)g^*(\alpha_{B2})}-\\ &\gamma_1\frac{\left[1-g^*(\alpha_{A1}+\alpha_{B1})\right]f^*(s)}{1-f^*(s)g^*(\alpha_{A1}+\alpha_{B1})}-\gamma_2\frac{\left[1-g^*(\alpha_{A2}+\alpha_{B2})\right]f^*(s)}{1-f^*(s)g^*(\alpha_{A2}+\alpha_{B2})}\end{aligned} \tag{17}$$

From (5) and (17) it can be shown that

$$E[T]=(M_1)\left[\begin{array}{l}\frac{\gamma_3}{1-g^*(\alpha_{A1}+\alpha_{B2})}+\frac{\gamma_4}{1-g^*(\alpha_{A2}+\alpha_{B1})}+\frac{\gamma_5}{1-g^*(\alpha_{A1})}+\\ \frac{\gamma_7}{1-g^*(\alpha_{B1})}-\frac{\gamma_6}{1-g^*(\alpha_{A2})}-\frac{\gamma_8}{1-g^*(\alpha_{B2})}-\\ \frac{\gamma_1}{1-g^*(\alpha_{A1}+\alpha_{B1})}-\frac{\gamma_2}{1-g^*(\alpha_{A2}+\alpha_{B2})}\end{array}\right] \tag{18}$$

and

$$\begin{aligned}E[T^2]=&2(M_2)\left[\begin{array}{l}\frac{\gamma_3}{1-g^*(\alpha_{A1}+\alpha_{B2})}+\frac{\gamma_4}{1-g^*(\alpha_{A2}+\alpha_{B1})}+\frac{\gamma_5}{1-g^*(\alpha_{A1})}+\\ \frac{\gamma_7}{1-g^*(\alpha_{B1})}-\frac{\gamma_6}{1-g^*(\alpha_{A2})}-\frac{\gamma_8}{1-g^*(\alpha_{B2})}-\\ \frac{\gamma_1}{1-g^*(\alpha_{A1}+\alpha_{B1})}-\frac{\gamma_2}{1-g^*(\alpha_{A2}+\alpha_{B2})}\end{array}\right]+\\ &2(M_1)^2\left[\begin{array}{l}\frac{\gamma_3 g^*(\alpha_{A1}+\alpha_{B2})}{(1-g^*(\alpha_{A1}+\alpha_{B2}))^2}+\frac{\gamma_4 g^*(\alpha_{A2}+\alpha_{B1})}{1(-g^*(\alpha_{A2}+\alpha_{B1}))^2}+\\ \frac{\gamma_5 g^*(\alpha_{A1})}{(1-g^*(\alpha_{A1}))^2}+\frac{\gamma_7 g^*(\alpha_{B1})}{(1-g^*(\alpha_{B1}))^2}-\frac{\gamma_6 g^*(\alpha_{A2})}{(1-g^*(\alpha_{A2}))^2}-\\ \frac{\gamma_8 g^*(\alpha_{B2})}{(1-g^*(\alpha_{B2}))^2}-\frac{\gamma_1 g^*(\alpha_{A1}+\alpha_{B1})}{(1-g^*(\alpha_{A1}+\alpha_{B1}))^2}-\\ \frac{\gamma_2 g^*(\alpha_{A2}+\alpha_{B2})}{(1-g^*(\alpha_{A2}+\alpha_{B2}))^2}\end{array}\right]\end{aligned} \tag{19}$$

Note 3.

The inter-decision times U_i form a geometric process with parameter ‘a’. Proceeding as in Model-I it is found that

$$E[T]=a\,(M_1)\left[\begin{array}{l}\frac{\gamma_3}{a-g^*(\alpha_{A1}+\alpha_{B2})}+\frac{\gamma_4}{a-g^*(\alpha_{A2}+\alpha_{B1})}+\frac{\gamma_5}{a-g^*(\alpha_{A1})}+\\ \frac{\gamma_7}{a-g^*(\alpha_{B1})}-\frac{\gamma_6}{a-g^*(\alpha_{A2})}-\frac{\gamma_8}{a-g^*(\alpha_{B2})}-\\ \frac{\gamma_1}{a-g^*(\alpha_{A1}+\alpha_{B1})}-\frac{\gamma_2}{a-g^*(\alpha_{A2}+\alpha_{B2})}\end{array}\right] \tag{20}$$

and

$$E[T^2]=2a^2[M_2]\begin{bmatrix}\frac{\gamma_3}{a^2-g^*(\alpha_{A1}+\alpha_{B2})}+\frac{\gamma_4}{a^2-g^*(\alpha_{A2}+\alpha_{B1})}+\\ \frac{\gamma_5}{a^2-g^*(\alpha_{A1})}+\frac{\gamma_7}{a^2-g^*(\alpha_{B1})}-\\ \frac{\gamma_6}{a^2-g^*(\alpha_{A2})}-\frac{\gamma_8}{a^2-g^*(\alpha_{B2})}-\\ \frac{\gamma_1}{a^2-g^*(\alpha_{A1}+\alpha_{B1})}-\frac{\gamma_2}{a^2-g^*(\alpha_{A2}+\alpha_{B2})}\end{bmatrix}-(M_1)^2\times$$

$$\sum_{k=0}^{\infty}\left[\left(\sum_{i=1}^{k}\frac{1}{a^{i-1}}\right)^2-\left(\sum_{i=1}^{k+1}\frac{1}{a^{i-1}}\right)^2\right]\begin{pmatrix}\gamma_3\left(g^*(\alpha_{A1}+\alpha_{B2})\right)^k+\gamma_5\left(g^*(\alpha_{A1})\right)^k+\\ \gamma_4\left(g^*(\alpha_{A2}+\alpha_{B1})\right)^k+\gamma_7\left(g^*(\alpha_{B1})\right)^k-\\ \gamma_6\left(g^*(\alpha_{A2})\right)^k-\gamma_8\left(g^*(\alpha_{B2})\right)^k-\\ \gamma_1\left(g^*(\alpha_{A1}+\alpha_{B1})\right)^k-\\ \gamma_2\left(g^*(\alpha_{A2}+\alpha_{B2})\right)^k\end{pmatrix}\quad(21)$$

Note 4.

The inter-decision times U_i are exchangeable and constantly correlated exponential random variables.

Proceeding as in Model-I it is shown that

$$E[T]=v\times\begin{bmatrix}\frac{\gamma_3}{1-g^*(\alpha_{A1}+\alpha_{B2})}+\frac{\gamma_4}{1-g^*(\alpha_{A2}+\alpha_{B1})}+\\ \frac{\gamma_5}{1-g^*(\alpha_{A1})}+\frac{\gamma_7}{1-g^*(\alpha_{B1})}-\frac{\gamma_6}{1-g^*(\alpha_{A2})}-\\ \frac{\gamma_8}{1-g^*(\alpha_{B2})}-\frac{\gamma_1}{1-g^*(\alpha_{A1}+\alpha_{B1})}-\frac{\gamma_2}{1-g^*(\alpha_{A2}+\alpha_{B2})}\end{bmatrix}\quad(22)$$

$$E[T^2]=2v^2\begin{bmatrix}\frac{\gamma_3(1+R^2g^*(\alpha_{A1}+\alpha_{B2}))}{\left(1-g^*(\alpha_{A1}+\alpha_{B2})\right)^2}+\frac{\gamma_4(1+R^2g^*(\alpha_{A2}+\alpha_{B1}))}{1-g^*(\alpha_{A2}+\alpha_{B1})}+\\ \frac{\gamma_5(1+R^2g^*(\alpha_{A1}))}{1-g^*(\alpha_{A1})}+\frac{\gamma_7(1+R^2g^*(\alpha_{B1}))}{1-g^*(\alpha_{B1})}-\\ \frac{\gamma(1+R^2g^*(\alpha_{A2}))_6}{1-g^*(\alpha_{A2})}-\frac{\gamma_8(1+R^2g^*(\alpha_{B2}))}{1-g^*(\alpha_{B2})}-\\ \frac{\gamma_1(1+R^2g^*(\alpha_{A1}+\alpha_{B1}))}{1-g^*(\alpha_{A1}+\alpha_{B1})}-\frac{\gamma_2(1+R^2g^*(\alpha_{A2}+\alpha_{B2}))}{1-g^*(\alpha_{A2}+\alpha_{B2})}\end{bmatrix}\quad(23)$$

IV. MODEL DESCRIPTION AND ANALYSIS FOR MODEL-3

For this Model, $Y=Y_A+Y_B$. All the other assumptions and notations are as in Model-I. Proceeding as in Model-I it can be shown that

$$P(T>t)=\gamma_9\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{A1})\right]^k+\gamma_{10}\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{B1})\right]^k-\gamma_{11}\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{A2})\right]^k-\gamma_{12}\sum_{k=1}^{\infty}[F_k(t)-F_{k+1}(t)]\left[g^*(\alpha_{B2})\right]^k\quad(24)$$

where

$$\gamma_9=\frac{\alpha_{A1}\alpha_{A2}\alpha_{B1}\alpha_{B2}}{(\alpha_{A2}-\alpha_{A1})(\alpha_{B1}-\alpha_{A1})(\alpha_{B2}-\alpha_{A1})},\gamma_{10}=\frac{\alpha_{A1}\alpha_{A2}\alpha_{B1}\alpha_{B2}}{(\alpha_{B2}-\alpha_{B1})(\alpha_{B1}-\alpha_{A1})(\alpha_{B1}-\alpha_{A2})},$$

$$\gamma_{11}=\frac{\alpha_{A1}\alpha_{A2}\alpha_{B1}\alpha_{B2}}{(\alpha_{A2}-\alpha_{A1})(\alpha_{B1}-\alpha_{A2})(\alpha_{B2}-\alpha_{A2})}\ and\ \frac{\alpha_{A1}\alpha_{A2}\alpha_{B1}\alpha_{B2}}{(\alpha_{B2}-\alpha_{B1})(\alpha_{B2}-\alpha_{A1})(\alpha_{B2}-\alpha_{A2})}$$

Proceeding as in Model-I we get

$$\ell^*(s)=\gamma_9\frac{\left[1-g^*(\alpha_{A1})\right]f^*(s)}{1-f^*(s)g^*(\alpha_{A1})}+\gamma_{10}\frac{\left[1-g^*(\alpha_{B1})\right]f^*(s)}{1-f^*(s)g^*(\alpha_{B1})}-\gamma_{11}\frac{\left[1-g^*(\alpha_{A2})\right]f^*(s)}{1-f^*(s)g^*(\alpha_{A2})}-\gamma_{12}\frac{\left[1-g^*(\alpha_{B2})\right]f^*(s)}{1-f^*(s)g^*(\alpha_{B2})}\quad(25)$$

From (5) and (25) it can be shown that

$$E[T]=(M_1)\begin{bmatrix}\frac{\gamma_9}{1-g^*(\alpha_{A1})}+\frac{\gamma_{10}}{1-g^*(\alpha_{B1})}-\\ \frac{\gamma_{11}}{1-g^*(\alpha_{A2})}-\frac{\gamma_{12}}{1-g^*(\alpha_{B2})}\end{bmatrix}\quad(26)$$

and

$$E[T^2]=2(M_2)\begin{bmatrix}\frac{\gamma_9}{1-g^*(\alpha_{A1})}+\frac{\gamma_{10}}{1-g^*(\alpha_{B1})}-\\ \frac{\gamma_{11}}{1-g^*(\alpha_{A2})}-\frac{\gamma_{12}}{1-g^*(\alpha_{B2})}\end{bmatrix}-2(M_1)^2\begin{bmatrix}\frac{\gamma_9g^*(\alpha_{A1})}{\left(1-g^*(\alpha_{A1})\right)^2}+\frac{\gamma_{10}g^*(\alpha_{B1})}{\left(1-g^*(\alpha_{B1})\right)^2}-\\ \frac{\gamma_{11}g^*(\alpha_{A2})}{\left(1-g^*(\alpha_{A2})\right)^2}-\frac{\gamma_{12}g^*(\alpha_{B2})}{\left(1-g^*(\alpha_{B2})\right)^2}\end{bmatrix}\quad(27)$$

Note 5.

The inter-decision times U_i form a geometric process with parameter ‘a’. Proceeding as in Model-I it is found that

Proceeding as in Model-I it is found that

$$E[T]=a\,(M_1)\begin{bmatrix}\frac{\gamma_9}{a-g^*(\alpha_{A1})}+\frac{\gamma_{10}}{a-g^*(\alpha_{B1})}+\\ \frac{\gamma_{11}}{a-g^*(\alpha_{A2})}+\frac{\gamma_{12}}{a-g^*(\alpha_{B2})}\end{bmatrix}\quad(28)$$

and

$$E[T^2]=2a^2[M_2]\left[\frac{\gamma_9}{a-g^*(\alpha_{A1})}+\frac{\gamma_{10}}{a-g^*(\alpha_{B1})}+\frac{\gamma_{11}}{a-g^*(\alpha_{A2})}+\frac{\gamma_{12}}{a-g^*(\alpha_{B2})}\right]-$$

$$(M_1)^2\left(\sum_{k=0}^{\infty}\left[\left(\sum_{i=1}^{k}\frac{1}{a^{i-1}}\right)^2-\left(\sum_{i=1}^{k+1}\frac{1}{a^{i-1}}\right)^2\right]\right)\times\begin{pmatrix}\gamma_9\left(g^*(\alpha_{A1})\right)^k+\gamma_{10}\left(g^*(\alpha_{B1})\right)^k-\\ \gamma_{11}\left(g^*(\alpha_{A2})\right)^k-\gamma_{12}\left(g^*(\alpha_{B2})\right)^{k^k}\end{pmatrix}\quad(29)$$

Note 6.

The inter-decision times U_i are exchangeable and constantly correlated exponential random variables.

Proceeding as in Model-I it is shown that

$$E[T] = v \times \left[\frac{\gamma_9}{1-g^*(\alpha_{A1})} + \frac{\gamma_{10}}{1-g^*(\alpha_{B1})} - \frac{\gamma_{11}}{1-g^*(\alpha_{A2})} - \frac{\gamma_{12}}{1-g^*(\alpha_{B2})} \right] \quad (30)$$

and

$$E[T^2] = 2 \times v^2 \left[\begin{array}{l} \frac{\gamma_9\left(1+R^2 g^*(\alpha_{A1})\right)}{1-g^*(\alpha_{A1})} + \frac{\gamma_{10}\left(1+R^2 g^*(\alpha_{B1})\right)}{1-g^*(\alpha_{B1})} - \\ \frac{\gamma_{11}\left(1+R^2 g^*(\alpha_{A2})\right)}{1-g^*(\alpha_{A2})} - \frac{\gamma_{12}\left(1+R^2 g^*(\alpha_{B2})\right)}{1-g^*(\alpha_{B2})} \end{array} \right] \quad (31)$$

V. NUMERICAL ILLUSTRATION

The influence of parameters on the performance measure namely mean and variance of the time for recruitment is studied numerically. In the following table these performance measures are calculated by varying the parameter ‘c’ and keeping the parameters α_{A1}, α_{A2}, α_{B1}, α_{B2} and λ fixed. $\alpha_{A1}=0.2$, $\alpha_{A2}=0.3$, $\alpha_{B1}=0.4$, $\alpha_{B2}=0.5$, k=2and $\lambda=0.75$

c	Model-I		Model-II		Model-III	
	$E(T)$	$V(T)$	$E(T)$	$V(T)$	$E(T)$	$V(T)$
3	3.33	11.11	10	71.56	12.67	96.44
4	4.22	17.83	13.11	121.33	16.67	164
5	5.11	26.12	16.22	184.15	20.67	249.33

Table: Effect of ‘c’ on the performance measures E[T] and V[T]

From the above table the following observation is given: As ‘c’ increases both E(T) and V(T) increases for all the three models.

VI. CONCLUSION

Model-3 is more suitable from the organization point of view as it postponed the time to recruitment compared to Models 1 and 2.

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