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# $\Lambda_\delta^s$-Separation axioms in bitopological spaces

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***Abstract*** **- The aim of this paper is to introduce the concept of $ij-\Lambda_\delta^s$ open sets and associated closure operator in bitopological spaces and we study some of the fundamental properties of such sets. Also we shall introduce the notions of pairwise $\Lambda_\delta^s-T_i$ and pairwise $\Lambda_\delta^s-R_i$ bitopological spaces for $i=0,1,2$ and investigate their properties.**

***Key words:*** ***$ij$ – $\delta$ open set, $ij-\delta$ semi open set, $ij-\Lambda_\delta^s$ open set, pairwise $\Lambda_\delta^s-T_0$, pairwise $\Lambda_\delta^s-T_1$, pairwise $\Lambda_\delta^s-T_2$, pairwise $\Lambda_\delta^s-R_0$, pairwise $\Lambda_\delta^s-R_1$.***

***AMS Subject Classification: 54A10, 54A25, 54D10, 54E55.***

## I. INTRODUCTION

In topology, the class of generalized Λ-sets studied by Maki in [16] and defined the associated closure operator $C^\Lambda$. El-Sharkasy [9] studied the concept of $\Lambda_\alpha-$sets and the associated topology $T^{\Lambda_\alpha}$. Caldas et al.[4,5] introduced the concept $\Lambda_\delta^s-$ sets (resp. $V_\delta^s-$ sets) in topological spaces, which is the intersection of $\delta-$semiopen (resp. union of $\delta-$semiclosed) sets. Khedr and Al-saadi [15] introduced and studied the concept of *ij*-*s*Λ -semi θ-closed and pairwise θ-generalized *s*Λ-set in bitopological spaces, which is an extension of the class of generalized Λ-sets. Ghareeb and Noiri [10] introduced the concept of Λ - Generalized closed sets in bitopological spaces. In 2006, Caldas et al. [6] introduced the notions of $\Lambda_\delta-T_0$, $\Lambda_\delta-T_1$, $\Lambda_\delta-R_0$ and $\Lambda_\delta-R_1$ in bitopological spaces. Quite recently, Edward Samuel and Balan [8] studied the concept of $\Lambda_\delta^s-$Sets in bitopological spaces.

The purpose of this paper is to continue research along these directions but this time by utilizing $ij-\Lambda_\delta^s$ open sets. In this paper, we introduce $ij-\Lambda_\delta^s$ open sets and associated closure operator in bitopological spaces and we study some of their fundamental properties. Also, we introduce the notions of $\Lambda_\delta^s-T_0$, $\Lambda_\delta^s-T_1$, $\Lambda_\delta^s-T_2$, $\Lambda_\delta^s-R_0$, $\Lambda_\delta^s-R_1$ bitopological spaces and the major properties of this new concept will be studied.

## II. PRELIMINARIES

Throughout the present paper, $(X,\tau_1,\tau_2)$ (or briefly X) always mean a bitopological space. Also $i,j=1,2$ and $i\neq j$. Let $A$ be a subset of $(X,\tau_1,\tau_2)$. By $i-int(A)$ and $i-cl(A)$, we mean respectively the interior and the closure of $A$ in the topological space $(X,\tau_i)$ for $i=1,2$. A subset A of X is called $ij-$regular open [12] if $A=i-int[j-cl(A)]$. A point *x* of X is called an $ij-\delta-$cluster point of A if

$i-Int(j-Cl(U))\cap A\neq\phi$ for every $\tau_i-$open set U containing $x$.

The set of all $ij-\delta-$cluster points of A is called the $ij-\delta-$ closure of A and is denoted by $ij-\delta Cl(A)$.

**Definition 2.1**[13] A subset A is said to be $ij-\delta$ closed if $ij-\delta cl(A)=A$. The complement of $ij-\delta$ closed set is said to be $ij-\delta$ open. The set of all $ij-\delta$ open (resp. $ij-\delta$ closed) sets of X will be denoted by $ij-\delta O(X)$( resp. $ij-\delta C(X)$).

**Definition 2.2**[7] A subset A of a bitopological space $(X,\tau_1,\tau_2)$ is called $ij-\delta$ semi open if there exists an $ij-\delta$ open set U such that $U\subseteq A\subseteq j-cl(U)$.

**Definition 2.3**[8] For a subset A of a bitopological space $(X,\tau_1,\tau_2)$, we define $A^{\delta s\Lambda_{ij}}$ and $A^{\delta sV_{ij}}$ as follows, $A^{\delta s\Lambda_{ij}}=\cap\{U: A\subseteq U, U\in ij-\delta SO(X)\}$ and $A^{\delta sV_{ij}}=\cup\{U: U\subseteq A, U^C\in ij-\delta SO(X)\}$.

**Definition 2.4**[8] A subset A of a bitopological space $(X,\tau_1,\tau_2)$ is called,

(a) $ij-\Lambda_\delta^s$ set if $A=A^{\delta s\Lambda_{ij}}$.
(b) $ij-V_\delta^s$ set if $A=A^{\delta sV_{ij}}$.

The family of all $ij-\Lambda_\delta^s$ sets (resp. $ij-V_\delta^s$) is denoted by $ij-\Lambda_\delta^s(X,\tau_1,\tau_2)$ (resp. $ij-V_\delta^s(X,\tau_1,\tau_2)$).

## III. $ij-\Lambda_\delta^s$ CLOSURE OPERATOR

**Definition 3.1** Let A be a subset of a bitopological space $(X,\tau_1,\tau_2)$,

(a) A is called a $ij-\Lambda_\delta^s$ closed set if $A=T\cap C$, where T is a $ij-\Lambda_\delta^s$ set and C is a $ji-\delta$ semi closed set. The complement of a $ij-\Lambda_\delta^s$ closed set is called $ij-\Lambda_\delta^s$ open. The family of all $ij-\Lambda_\delta^s$ open sets and $ij-\Lambda_\delta^s$ closed sets are denoted by $ij-\Lambda_\delta^s O(X,\tau_1,\tau_2)$ and $ij-\Lambda_\delta^s C(X,\tau_1,\tau_2)$.

(b) A point $x\in(X,\tau_1,\tau_2)$ is called a $ij-\Lambda_\delta^s$ cluster point of A if for every $ij-\Lambda_\delta^s$ open set U of $(X,\tau_1,\tau_2)$ containing $x$, $A\cap U\neq\phi$. The set of all $ij-\Lambda_\delta^s$ cluster points is called the $ij-\Lambda_\delta^s$ closure of A and is denoted by $ij-C^{\Lambda_\delta^s}(A)$.

**Theorem 3.2** Let A, B and $\{B_\alpha,\alpha\in J\}$ be subsets of a bitopological space $(X,\tau_1,\tau_2)$. For $ij-\Lambda_\delta^s$ closure, the following properties hold,

(a) $A\subseteq ij-C^{\Lambda_\delta^s}(A)$.
(b) $ij-C^{\Lambda_\delta^s}(A)=\{U: A\subseteq U, U\in ij-\Lambda_\delta^s C(X,\tau_1,\tau_2)\}$.
(c) If $A\subseteq B$, then $ij-C^{\Lambda_\delta^s}(A)\subseteq ij-C^{\Lambda_\delta^s}(B)$.

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(d) $A$ is $ij-\Lambda_\delta^s$ closed if and only if $A = ij - C^{\Lambda_\delta^s}(A)$.
(e) $ij - C^{\Lambda_\delta^s}(A)$ is $ij-\Lambda_\delta^s$ closed.
(f) $\bigcup_{\alpha\in J} ij - C^{\Lambda_\delta^s}(B_\alpha) = ij - C^{\Lambda_\delta^s}(\bigcup_{\alpha\in J} B_\alpha)$.
(g) $ij - C^{\Lambda_\delta^s}[ij - C^{\Lambda_\delta^s}(A)] = ij - C^{\Lambda_\delta^s}(A)$.

***Proof.*** (a), (b), (c), (e) Obvious. From the definition.

(d) Obvious. From (a) and definition.

(f) Suppose that there exists a point $x$ such that $x \notin ij - C^{\Lambda_\delta^s}(\bigcup_{\alpha\in J} B_\alpha)$. Then, there exists a subset $U \in ij - \Lambda_\delta^s(X,\tau_1,\tau_2)$ such that $\bigcup_{\alpha\in J} B_\alpha \subseteq U$ and $x \notin U$. Thus for each $\alpha \in J$, we have $x \notin ij - C^{\Lambda_\delta^s}(B_\alpha)$. Thus implies that $x \notin \bigcup_{\alpha\in J} ij - C^{\Lambda_\delta^s}(B_\alpha)$.

Conversely, we suppose that there exists a point $x \in X$ such that $x \notin \bigcup_{\alpha\in J} ij - C^{\Lambda_\delta^s}(B_\alpha)$. Then, there exist subsets $U_\alpha \in ij - \Lambda_\delta^s(X,\tau_1,\tau_2)$ for each $\alpha \in J$ such that $x \notin U_\alpha$ and $B_\alpha \subseteq U_\alpha$. Let $U = \bigcup_{\alpha\in J} U_\alpha$, we have $x \notin U$, $\bigcup_{\alpha\in J} B_\alpha \subseteq U_\alpha$ and $U \in ij - \Lambda_\delta^s(X,\tau_1,\tau_2)$. Thus $x \notin ij - C^{\Lambda_\delta^s}(\bigcup_{\alpha\in J} B_\alpha)$.

(g) Suppose that there exists a point $x \in X$ such that $x \notin ij - C^{\Lambda_\delta^s}(A)$. Then there exists a subset $U \in ij - \Lambda_\delta^s(X,\tau_1,\tau_2)$ such that $x \notin U$ and $U \supseteq A$. Since $U \in ij - \Lambda_\delta^s(X,\tau_1,\tau_2)$ we have $ij - C^{\Lambda_\delta^s}(A) \subseteq U$. Thus we have $x \notin ij - C^{\Lambda_\delta^s}[ij - C^{\Lambda_\delta^s}(A)]$. Therefore $ij - C^{\Lambda_\delta^s}[ij - C^{\Lambda_\delta^s}(A)] \subseteq ij - C^{\Lambda_\delta^s}(A)$.

**Theorem 3.3** Let A be a subset of a bitopological space $(X,\tau_1,\tau_2)$, then following are hold,

(a) If $A_\alpha$ is $ij-\Lambda_\delta^s$ closed sets for each $\alpha \in J$, then $\bigcap_{\alpha\in J} U_\alpha$ is $ij-\Lambda_\delta^s$ closed.
(b) If $A_\alpha$ is $ij-\Lambda_\delta^s$ open sets for each $\alpha \in J$, then $\bigcup_{\alpha\in J} U_\alpha$ is $ij-\Lambda_\delta^s$ open.

**Proof.** Obvious.

**Definition 3.4** Let A be a subset of a bitopological space $(X,\tau_1,\tau_2)$, then the $ij-\Lambda_\delta^s$ kernel of A, denoted by $ij-\Lambda_\delta^s\mathrm{Ker}(A)$ is defined to be the set $ij-\Lambda_\delta^s\mathrm{Ker}(A) = \bigcap\{U \in ij - \Lambda_\delta^s O(X,\tau_1,\tau_2) : A \subseteq U\}$.

**Theorem 3.5** For any two subsets A and B of a bitopological space $(X,\tau_1,\tau_2)$,

(a) If $A \subseteq B$, then $ij-\Lambda_\delta^s\mathrm{Ker}(A) \subseteq ij-\Lambda_\delta^s\mathrm{Ker}(A)$.
(b) $ij-\Lambda_\delta^s\mathrm{Ker}[ij-\Lambda_\delta^s\mathrm{Ker}(A)] = ij-\Lambda_\delta^s\mathrm{Ker}(A)$.

***Proof.*** Obvious.

**Theorem 3.6** For any two points $x$ and $y$ of a bitopological space $(X,\tau_1,\tau_2)$, $y \in ij-\Lambda_\delta^s\mathrm{Ker}(\{x\})$ if and only if $x \in ij - C^{\Lambda_\delta^s}(\{y\})$.

***Proof.*** Let $y \notin ij-\Lambda_\delta^s\mathrm{Ker}(\{x\})$. Then there exists a $ij-\Lambda_\delta^s$ open set U containing $x$ such that $y \notin U$. Hence $x \notin ij - C^{\Lambda_\delta^s}(\{y\})$. Similarly the converse is true.

**Theorem 3.7** If $(X,\tau_1,\tau_2)$ be a bitopological space and $A \subset X$, then $ij-\Lambda_\delta^s\mathrm{Ker}(A) = \{x \in X : ij - C^{\Lambda_\delta^s}(\{x\}) \cap A \neq \phi\}$.

***Proof.*** Let $x \in ij-\Lambda_\delta^s\mathrm{Ker}(A)$ and suppose that $ij - C^{\Lambda_\delta^s}(\{x\}) \cap A = \phi$. Then $x \notin X \backslash ij - C^{\Lambda_\delta^s}(\{x\})$ which is a $ij-\Lambda_\delta^s$ open set containing A. This is impossible, since $x \in ij-\Lambda_\delta^s\mathrm{Ker}(A)$. Consequently, $ij - C^{\Lambda_\delta^s}(\{x\}) \cap A \neq \phi$. Next, let $x \in X$ such that $ij - C^{\Lambda_\delta^s}(\{x\}) \cap A \neq \phi$ and suppose that $x \notin ij-\Lambda_\delta^s\mathrm{Ker}(A)$. Then there exists a $ij-\Lambda_\delta^s$ open set U containing A and $x \notin U$. Let $y \in ij - C^{\Lambda_\delta^s}(\{x\}) \cap A$. Hence U is a $ij-\Lambda_\delta^s$ neigbourhood of y which does not contain $x$. By this contradiction $x \in ij-\Lambda_\delta^s\mathrm{Ker}(A)$.

**Definition 3.8** A bitopological space space $(X,\tau_1,\tau_2)$ is called,

(a) pairwise $\Lambda_\delta^s - T_0$ if for each pair of distinct points in $X$, there is a $ij-\Lambda_\delta^s$ open set containing one of the points but not the other.

(b) pairwise $\Lambda_\delta^s - T_1$ if for each pair of distinct points $x$ and $y$ in $X$, there is a $ij-\Lambda_\delta^s$ open $U$ in $X$ containing $x$ but not $y$ and a $ji-\Lambda_\delta^s$ open set $V$ in $X$ containing $y$ but not $x$.

(c) pairwise $\Lambda_\delta^s - T_2$ if for each pair of distinct points $x$ and $y$ in $X$, there exist a $ij-\Lambda_\delta^s$ open set $U$ and $ji-\Lambda_\delta^s$ open set $V$ such that $x \in U$, $y \in V$ and $U \cap V = \phi$.

**Remark 3.9** If a bitopological space space $(X,\tau_1,\tau_2)$ is pairwise $\Lambda_\delta^s - T_i$, then it is pairwise $\Lambda_\delta^s - T_{i-1}$, $i = 1, 2$.

**Theorem 3.10** A bitopological space space $(X,\tau_1,\tau_2)$ is pairwise $\Lambda_\delta^s - T_0$ if and only if for each pair of distinct points $x, y$ of $X$, $ij - C^{\Lambda_\delta^s}(\{x\}) \neq ji - C^{\Lambda_\delta^s}(\{y\})$.

***Proof.*** Suppose that $x, y \in X$, $x \neq y$ and $ij - C^{\Lambda_\delta^s}(\{x\}) \neq ji - C^{\Lambda_\delta^s}(\{y\})$. Let $z$ be a point of $X$ such that $z \in ij - C^{\Lambda_\delta^s}(\{x\})$ but $z \notin ji - C^{\Lambda_\delta^s}(\{y\})$. We claim that $x \notin ji - C^{\Lambda_\delta^s}(\{y\})$. For it, if $x \in ji - C^{\Lambda_\delta^s}(\{y\})$ then $ij - C^{\Lambda_\delta^s}(\{x\}) \subseteq ji - C^{\Lambda_\delta^s}(\{y\})$ and this contradicts the fact that $z \notin ji - C^{\Lambda_\delta^s}(\{y\})$. Consequently, $x \in ji - \Lambda_\delta^s O(X,\tau_1,\tau_2)$, $[ji - C^{\Lambda_\delta^s}(\{y\})]^C$ to which $y$ does not belong.

Conversely, Let $(X,\tau_1,\tau_2)$ be a pairwise $\Lambda_\delta^s - T_0$ space and $x, y$ be any two distinct points of $X$. There exists a $ij-\Lambda_\delta^s$ open set $G$ containing $x$ or $y$, say $x$ but not $y$. Then $G^C$ is a $ij-\Lambda_\delta^s$ closed set which does not contain $x$ but contains $y$. Since $ji - C^{\Lambda_\delta^s}(\{y\})$ is the smallest $ji-\Lambda_\delta^s$ closed set containing $y$, $ji - C^{\Lambda_\delta^s}(\{y\}) \subseteq G^C$, and so $x \notin ji - C^{\Lambda_\delta^s}(\{y\})$. Consequently, $ij - C^{\Lambda_\delta^s}(\{x\}) \neq ji - C^{\Lambda_\delta^s}(\{y\})$.

**Theorem 3.11** A bitopological space $(X,\tau_1,\tau_2)$ is pairwise $\Lambda_\delta^s - T_1$ if and only if the singletons are $ij-\Lambda_\delta^s$ closed sets.

***Proof.*** Suppose that $(X,\tau_1,\tau_2)$ is pairwise $\Lambda_\delta^s - T_1$ and $x$ be any point of $X$. Let $y \in \{x\}^C$. Then $x \neq y$ and so there exists a $ij-\Lambda_\delta^s$ open set $U_y$ such that $y \in U_y$ but $x \notin U_y$. Consequently, $y \in U_y \subseteq \{x\}^C$ i.e., $\{x\}^C = \bigcup\{U_y : y \in \{x\}^C\}$ which is $ij-\Lambda_\delta^s$ open.

Conversely, suppose that $\{p\}$ is $ij-\Lambda_\delta^s$ closed for every $p \in X$. Let $x, y \in X$ with $\neq y$. Now $x \neq y$ implies $y \in \{x\}^C$. Hence $\{x\}^C$ is a $ij-\Lambda_\delta^s$ open set containing $y$ but not containing $x$. Similarly $\{y\}^C$ is a $ji-\Lambda_\delta^s$ open set containing $x$ but not $y$. Therefore, $(X,\tau_1,\tau_2)$ is a pairwise $\Lambda_\delta^s - T_1$ space.

**Definition 3.12** A bitopological space $(X,\tau_1,\tau_2)$ is pairwise $\Lambda_\delta^s -$ symmetric if for $x$ and $y$ in $X$, $x \in ji - C^{\Lambda_\delta^s}(\{y\})$ implies $y \in ji - C^{\Lambda_\delta^s}(\{x\})$.

**Definition 3.13** A subset $A$ of a bitopological space $(X,\tau_1,\tau_2)$ is called a $ij-\Lambda_\delta^s$ generalized closed set (briefly $ij-\Lambda_\delta^s - g$

closed) if $ji - C^{\Lambda_\delta^s}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $ij - \Lambda_\delta^s$ open.

**Theorem 3.14** Every $ij - \Lambda_\delta^s$ closed set is $ij - \Lambda_\delta^s - \mathrm{g}$ closed.

**Remark 3.15** The converse of above theorem is not true in general.

**Theorem 3.16** A bitopological space $(X, \tau_1, \tau_2)$ is pairwise $\Lambda_\delta^s -$ symmetric if and only if $\{x\}$ is $ij - \Lambda_\delta^s - \mathrm{g}$ closed for each $x \in X$.

***Proof.*** Assume that $x \in ji - C^{\Lambda_\delta^s}(\{y\})$ but $y \notin ij - C^{\Lambda_\delta^s}(\{x\})$. This implies that the $[ij - C^{\Lambda_\delta^s}(\{x\})]^C$ contains $y$. Therefore, the set $\{y\}$ is a subset of $[ij - C^{\Lambda_\delta^s}(\{x\})]^C$. This implies that $ji - C^{\Lambda_\delta^s}(\{y\})$ is a subset of $[ij - C^{\Lambda_\delta^s}(\{x\})]^C$. Now $[ij - C^{\Lambda_\delta^s}(\{x\})]^C$ contains $x$ which is a contradiction.

Conversely, suppose that $\{x\} \subseteq U \in ij - \Lambda_\delta^s O(X, \tau_1, \tau_2)$, but $ij - C^{\Lambda_\delta^s}(\{x\})$ is not a subset of $U$. This means that $ij - C^{\Lambda_\delta^s}(\{x\})$ and $U^C$ are not disjoint. Let $y \in ij - C^{\Lambda_\delta^s}(\{x\}) \cap (X \backslash U)$. Now we have $x \in ji - C^{\Lambda_\delta^s}(\{y\})$ which is a subset of $U^C$ and $x \notin U$. This is a contradiction.

**Theorem 3.17** If a bitopological space $(X, \tau_1, \tau_2)$ is pairwise $\Lambda_\delta^s - T_1$ space, then it is pairwise

$\Lambda_\delta^s -$ symmetric.

***Proof.*** In a pairwise $\Lambda_\delta^s - T_1$ space, singleton sets are $ij - \Lambda_\delta^s$ closed and Therefore, $ij - \Lambda_\delta^s - \mathrm{g}$ closed. By theorem 3.16, $(X, \tau_1, \tau_2)$ is pairwise $\Lambda_\delta^s -$ symmetric.

**Theorem 3.18** For a bitopological space $(X, \tau_1, \tau_2)$ the following are equivalent:

(a) $(X, \tau_1, \tau_2)$ is pairwise $\Lambda_\delta^s -$ symmetric and pairwise $\Lambda_\delta^s - T_0$.

(b) $(X, \tau_1, \tau_2)$ is pairwise $\Lambda_\delta^s - T_1$.

***Proof.*** (a) $\Longrightarrow$(b) Let $x \neq y$ and by pairwise $\Lambda_\delta^s - T_0$, by remark 3.9 we may assume that $x \in U_1 \subseteq \{y\}^C$ for some $U_1 \in ij - \Lambda_\delta^s O(X, \tau_1, \tau_2)$. Then $x \notin ji - C^{\Lambda_\delta^s}(\{y\})$. Therefore, by the definition of pairwise $\Lambda_\delta^s -$ symmetric, we have $y \notin ij - C^{\Lambda_\delta^s}(\{x\})$. There exists a $U_2 \in ji - \Lambda_\delta^s O(X, \tau_1, \tau_2)$ such that $y \in U_2 \subseteq \{x\}^C$. Therefore, $(X, \tau_1, \tau_2)$ is a pairwise $\Lambda_\delta^s - T_1$ space.

**Theorem 3.19** For a pairwise $\Lambda_\delta^s -$ symmetric space $(X, \tau_1, \tau_2)$ the following are equivalent:

(1) $(X, \tau_1, \tau_2)$ is pairwise $\Lambda_\delta^s - T_0$.

(2) $(X, \tau_1, \tau_2)$ is pairwise $\Lambda_\delta^s - T_1$.

***Proof.*** (1) $\Longrightarrow$ (2) Obvious. From theorem 3.18.

(2) $\Longrightarrow$ (1) Obvious. From Remark 3.9.

## IV. PAIRWISE $\Lambda_\delta^s - R_0$ SPACES

**Definition 4.1** A bitopological space $(X, \tau_1, \tau_2)$ is a pairwise $\Lambda_\delta^s - R_0$ if for each $ij - \Lambda_\delta^s$ open set U, $x \in U$ implies $ji - C^{\Lambda_\delta^s}(\{x\}) \subseteq U$.

**Theorem 4.2** In a bitopological space $(X, \tau_1, \tau_2)$, the following statements are equivalent:

(a) $(X, \tau_1, \tau_2)$ is pairwise $\Lambda_\delta^s - R_0$.

(b) for any $ij - \Lambda_\delta^s$ closed set $G$ and a point $x \notin G$, there exists $U \in ji - \Lambda_\delta^s O(X, \tau_1, \tau_2)$ such that $x \notin U$ and $G \subseteq U$.

(c) for any $ij - \Lambda_\delta^s$ closed set $G$ and $x \notin G$, then $ji - C^{\Lambda_\delta^s}(\{x\}) \cap G = \phi$.

***Proof.*** (a) $\Longrightarrow$ (b): Let $G$ be a $ij - \Lambda_\delta^s$ closed set and $x \notin G$. Then by (a), $ji - C^{\Lambda_\delta^s}(\{x\}) \subseteq X \backslash G$. Let $U = X \backslash ji - C^{\Lambda_\delta^s}(\{x\})$, then $U \in ji - \Lambda_\delta^s O(X, \tau_1, \tau_2)$ and also $G \subseteq U$ and $x \notin U$.

(b) $\Longrightarrow$ (c): Let $G$ be a $ij - \Lambda_\delta^s$ closed set and a point $x \notin G$. Then by (b), there exists $U \in ji - \Lambda_\delta^s O(X, \tau_1, \tau_2)$ such that $G \subseteq U$ and $x \notin U$. Since $U \in ji - \Lambda_\delta^s O(X, \tau_1, \tau_2)$, $U \cap ji - C^{\Lambda_\delta^s}(\{x\}) = \phi$. Then $\cap\ ji - C^{\Lambda_\delta^s}(\{x\}) = \phi$ .

(c) $\Longrightarrow$ (a): Let $G \in ij - \Lambda_\delta^s O(X, \tau_1, \tau_2)$ and $x \in G$. Now $X \backslash G$ is $ij - \Lambda_\delta^s$ closed and $x \notin X \backslash G$ x. By (c), $ji - C^{\Lambda_\delta^s}(\{x\}) \cap (X \backslash G) = \phi$ and hence $ji - C^{\Lambda_\delta^s}(\{x\}) \subseteq G$ . Therefore, $(X, \tau_1, \tau_2)$ is pairwise $\Lambda_\delta^s - R_0$.

**Theorem 4.3** A bitopological space $(X, \tau_1, \tau_2)$ is pairwise $\Lambda_\delta^s - R_0$ if and only if for each pair $x$, $y$ of distinct points in $X$ , $ij - C^{\Lambda_\delta^s}(\{x\}) \cap ji - C^{\Lambda_\delta^s}(\{y\}) = \phi$ or $\{x, y\} \subseteq ij - C^{\Lambda_\delta^s}(\{x\}) \cap ji - C^{\Lambda_\delta^s}(\{y\})$.

***Proof.*** Let $(X, \tau_1, \tau_2)$ be pairwise $\Lambda_\delta^s - R_0$ . Suppose that $ij - C^{\Lambda_\delta^s}(\{x\}) \cap ji - C^{\Lambda_\delta^s}(\{y\}) \neq \phi$ and $\{x, y\} \nsubseteq ij - C^{\Lambda_\delta^s}(\{x\}) \cap ji - C^{\Lambda_\delta^s}(\{y\})$ . Let $p \in ij - C^{\Lambda_\delta^s}(\{x\}) \cap ji - C^{\Lambda_\delta^s}(\{y\})$ and $x \notin ij - C^{\Lambda_\delta^s}(\{x\}) \cap ji - C^{\Lambda_\delta^s}(\{y\})$ . Then $x \notin ji - C^{\Lambda_\delta^s}(\{y\})$ and $x \in X \backslash ji - C^{\Lambda_\delta^s}(\{y\}) \in ji - \Lambda_\delta^s O(X, \tau_1, \tau_2)$. But $ij - C^{\Lambda_\delta^s}(\{x\})$ is not a subset of $X \backslash ji - C^{\Lambda_\delta^s}(\{y\})$, this is a contradiction. Hence for each pair $x$, $y$ of distinct points in $X$, $ij - C^{\Lambda_\delta^s}(\{x\}) \cap ji - C^{\Lambda_\delta^s}(\{y\}) = \phi$ or $\{x, y\} \subseteq ij - C^{\Lambda_\delta^s}(\{x\}) \cap ji - C^{\Lambda_\delta^s}(\{y\})$.

Conversely, let $U$ be a $ij - \Lambda_\delta^s$ open set and $x \in U$. Suppose that $ji - C^{\Lambda_\delta^s}(\{x\})$ is not a subset of $U$. So there is a point $y \in ji - C^{\Lambda_\delta^s}(\{x\})$ such that $y \notin U$ and $-C^{\Lambda_\delta^s}(\{x\}) \cap U = \phi$ . Since $X \backslash U$ is $ij - \Lambda_\delta^s$ closed and $y \in X \backslash U$. Hence $\{x, y\} \nsubseteq ij - C^{\Lambda_\delta^s}(\{x\}) \cap ji - C^{\Lambda_\delta^s}(\{y\})$ and thus $ij - C^{\Lambda_\delta^s}(\{x\}) \cap ji - C^{\Lambda_\delta^s}(\{y\}) \neq \phi$.

**Theorem 4.4** In a bitopological space $(X, \tau_1, \tau_2)$, the following statements are equivalent:

(1) $(X, \tau_1, \tau_2)$ is pairwise $\Lambda_\delta^s - R_0$.

(2) For any $x \in X$, $ij - C^{\Lambda_\delta^s}(\{x\}) = ji - \Lambda_\delta^s \mathrm{Ker}(\{x\})$.

(3) For any $x \in X$, $ij - C^{\Lambda_\delta^s}(\{x\}) \subseteq ji - \Lambda_\delta^s \mathrm{Ker}(\{x\})$.

(4) For any $x, y \in X$, $y \in ij - C^{\Lambda_\delta^s}(\{x\})$ if and only if $x \in ji - C^{\Lambda_\delta^s}(\{y\})$.

(5) For any $ij - \Lambda_\delta^s$ closed set $F$ , $F = \cap\{G : G$ is a $ij - \Lambda_\delta^s$ open set and $F \subseteq G\}$.

(6) For any $ij - \Lambda_\delta^s$ open set $G$ , $G = \cup\{F : F$ is a $ij - \Lambda_\delta^s$ closed set and $F \subseteq G\}$.

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(7) For every $A \neq \phi$ and each $G \in ij - \Lambda_\delta^s O(X, \tau_1, \tau_2)$ such that $A \cap G \neq \phi$, there exists a $ji - \Lambda_\delta^s$ closed set $F$ such that $F \subseteq G$ and $A \cap F \neq \phi$.

***Proof.*** (1) ⟹ (2) Let $x, y \in X$. Then by theorem 3.6 and 4.3, $y \in ji - \Lambda_\delta^s \mathrm{Ker}(\{x\})$ , implies $x \in ji - C^{\Lambda_\delta^s}(\{y\})$ , $y \in ij - C^{\Lambda_\delta^s}(\{x\})$. Hence $ij - C^{\Lambda_\delta^s}(\{x\}) = ji - \Lambda_\delta^s \mathrm{Ker}(\{x\})$.

(2) ⟹ (3) Obvious.

(3) ⟹ (4) For any $x, y \in X$ , if $y \in ij - C^{\Lambda_\delta^s}(\{x\})$ , then $y \in ji - \Lambda_\delta^s \mathrm{Ker}(\{x\})$ by (3). Then by theorem 3.6, $x \in ji - C^{\Lambda_\delta^s}(\{y\})$. Similarly the converse.

(4) ⟹ (5) Let $F$ be a $ij - \Lambda_\delta^s$ closed set and $H = \bigcap\{G: G \text{ is a } ji - \Lambda_\delta^s \text{ open set and } F \subseteq G\}$. . Clearly $F \subseteq H$ . Let $x \notin F$. Then for any $y \in F$, we have that $ij - C^{\Lambda_\delta^s}(\{y\}) \subseteq F$ . Hence follows that $x \notin ij - C^{\Lambda_\delta^s}(\{y\})$ . Now by (4), $y \notin ji - C^{\Lambda_\delta^s}(\{x\})$. There exists a $ji - \Lambda_\delta^s$ open set $G_y$ such that $y \in G_y$ and $x \notin G_y$ . Let $G = \bigcup_{y \in F}\{G_y : G_y \text{ is a } ji - \Lambda_\delta^s \text{ open set}, y \in G_y \text{ and } x \notin G_y\}$. Thus, there exists a $ji - \Lambda_\delta^s$ open set $G$ such that $x \notin G$ and $F \subseteq G$ . Hence, $x \notin H$ . Therefore, $F = H$.

(5) ⟹ (6) Obvious.

(6) ⟹ (7) Let $A \neq \phi$ and $G$ be a $ij - \Lambda_\delta^s$ open set and $x \in A \cap G$ . By (6), $G = \bigcup\{F: F \text{ is a } ij - \Lambda_\delta^s \text{ closed set and } F \subseteq G\}$. It follows that there is a $ij - \Lambda_\delta^s$ closed set $F$ such that $x \in A \subseteq G$. Hence $A \cap F \neq \phi$.

(7) ⟹ (1) Let $G$ be a $ij - \Lambda_\delta^s$ open set and $x \in G$ , then $\{x\} \cap G \neq \phi$. Therefore by (7), there exists a $ji - \Lambda_\delta^s$ closed $F$ such that $x \in F \subseteq G$ and $\{x\} \cap F \neq \phi$ , which implies $ji - C^{\Lambda_\delta^s}(\{x\}) \subseteq G$. Therefore, $(X, \tau_1, \tau_2)$ is pairwise $\Lambda_\delta^s - R_0$.

**Theorem 4.5** In a bitopological space $(X, \tau_1, \tau_2)$, the following properties are equivalent:

(1) $(X, \tau_1, \tau_2)$ is pairwise $\Lambda_\delta^s - R_0$.

(2) For any $ij - \Lambda_\delta^s$ closed set $F \subset X$, $F = ji - \Lambda_\delta^s Ker(F)$.

(3) For any $ij - \Lambda_\delta^s$ closed set $F \subset X$ and $x \in F$ , $ji - \Lambda_\delta^s Ker(\{x\}) \subseteq F$.

(4) For any $x \in X$, $ji - \Lambda_\delta^s Ker(\{x\}) \subseteq ij - C^{\Lambda_\delta^s}(\{x\})$.

***Proof.*** (1) ⟹ (2) Let F be $ij - \Lambda_\delta^s$ closed and $x \notin F$. Then $X \backslash F$ is $ij - \Lambda_\delta^s$ open containing x. Since $(X, \tau_1, \tau_2)$ is pairwise $\Lambda_\delta^s - R_0$ , $ji - C^{\Lambda_\delta^s}(\{x\}) \subseteq X \backslash F$ . Therefore, $ji - C^{\Lambda_\delta^s}(\{x\}) \cap F = \phi$ and by theorem 3.7, $x \notin ji - \Lambda_\delta^s Ker(F)$ . Hence $F = ji - \Lambda_\delta^s Ker(F)$.

(2) ⟹ (3) Let $F$ be a $ij - \Lambda_\delta^s$ closed set containing $x$. Then $\{x\} \subseteq F$ and $ji - C^{\Lambda_\delta^s}(\{x\}) \subseteq ji - \Lambda_\delta^s Ker(F)$ . From (2), it follows that $ji - C^{\Lambda_\delta^s}(\{x\}) \subseteq F$.

(3) ⟹ (4) Since $x \in ij - C^{\Lambda_\delta^s}(\{x\})$ and $ij - C^{\Lambda_\delta^s}(\{x\})$ is $ij - \Lambda_\delta^s$ closed in X, by (3) it follows that $ji - \Lambda_\delta^s Ker(F) \subseteq ij - C^{\Lambda_\delta^s}(\{x\})$.

(4) ⟹ (1) Obvious. Proof follows from theorem 4.4.

**Remark 4.6** Let $(X, \tau_1, \tau_2)$ be a bitopolgical space. Then for each $x \in X$, let $bi - \Lambda_\delta^s(\{x\}) = 12 - C^{\Lambda_\delta^s}(\{x\}) \cap 21 - C^{\Lambda_\delta^s}(\{x\})$ and $bi - \Lambda_\delta^s \mathrm{Ker}(\{x\}) = 12 - \Lambda_\delta^s Ker(\{x\}) \cap 21 - \Lambda_\delta^s Ker(\{x\})$.

**Theorem 4.7** If a bitopological space $(X, \tau_1, \tau_2)$ is pairwise $\Lambda_\delta^s - R_0$ then for each pair of distinct points $x, y \in X$, either $bi - \Lambda_\delta^s(\{x\}) = bi - \Lambda_\delta^s(\{y\})$ or $bi - \Lambda_\delta^s(\{x\}) \cap bi - \Lambda_\delta^s(\{y\}) = \phi$.

***Proof.*** Let $(X, \tau_1, \tau_2)$ be a pairwise $\Lambda_\delta^s - R_0$ space. Suppose that $bi - \Lambda_\delta^s(\{x\}) \neq bi - \Lambda_\delta^s(\{y\})$ and $bi - \Lambda_\delta^s(\{x\}) \cap bi - \Lambda_\delta^s(\{y\}) \neq \phi$ . Let $s \in bi - \Lambda_\delta^s(\{x\}) \cap bi - \Lambda_\delta^s(\{y\})$ and $x \notin bi - \Lambda_\delta^s(\{y\}) = 12 - C^{\Lambda_\delta^s}(\{y\}) \cap 21 - C^{\Lambda_\delta^s}(\{y\})$ . Then $x \notin ij - C^{\Lambda_\delta^s}(\{y\})$ . And $x \in X \backslash ij - C^{\Lambda_\delta^s}(\{y\}) \in ij - \Lambda_\delta^s O(X, \tau_1, \tau_2)$. But $ji - C^{\Lambda_\delta^s}(\{x\})$ is not a subset of $X \backslash ij - C^{\Lambda_\delta^s}(\{y\})$ since $s \in bi - \Lambda_\delta^s(\{x\}) \cap bi - \Lambda_\delta^s(\{y\})$ . Thus $(X, \tau_1, \tau_2)$ is not a pairwise $\Lambda_\delta^s - R_0$ space which is a contradiction to our assumption. Hence we have either $bi - \Lambda_\delta^s(\{x\}) = bi - \Lambda_\delta^s(\{y\})$ or $bi - \Lambda_\delta^s(\{x\}) \cap bi - \Lambda_\delta^s(\{y\}) = \phi$.

## V. PAIRWISE $\Lambda_\delta^s - R_1$ SPACES

**Definition 5.1** A bitopological space $(X, \tau_1, \tau_2)$ is said to be pairwise $\Lambda_\delta^s - R_1$ if for each $x, y \in X$ , $ij - C^{\Lambda_\delta^s}(\{x\}) \neq ji - C^{\Lambda_\delta^s}(\{y\})$, there exist disjoint sets $U \in ji - \Lambda_\delta^s O(X, \tau_1, \tau_2)$ and $V \in ij - \Lambda_\delta^s O(X, \tau_1, \tau_2)$ such that $ij - C^{\Lambda_\delta^s}(\{x\}) \subseteq U$ and $ji - C^{\Lambda_\delta^s}(\{y\}) \subseteq V$.

**Theorem 5.2** If a bitopological space $(X, \tau_1, \tau_2)$ is pairwise $\Lambda_\delta^s - R_1$, then it is pairwise $\Lambda_\delta^s - R_0$.

***Proof.*** Suppose that $(X, \tau_1, \tau_2)$ is pairwise $\Lambda_\delta^s - R_1$. Let $U$ be a $ij - \Lambda_\delta^s$ open set and $x \in U$. Then for each point $y \in X \backslash U$, $ji - C^{\Lambda_\delta^s}(\{x\}) \neq ij - C^{\Lambda_\delta^s}(\{y\})$. Since $(X, \tau_1, \tau_2)$ is pairwise $\Lambda_\delta^s - R_1$, there exists a $ij - \Lambda_\delta^s$ open set $U_y$ and a $ji - \Lambda_\delta^s$ open set $V_y$ such that $ji - C^{\Lambda_\delta^s}(\{x\}) \subseteq U_y$ , $ij - C^{\Lambda_\delta^s}(\{y\}) \subseteq V_y$ and $U_y \cap V_y = \phi$. Let $A = \bigcup\{V_y : y \in X \backslash U\}$. Then $X \backslash U \subseteq A$, $x \notin A$ and $A$ is a $ji - \Lambda_\delta^s$ open set. Therefore, $ji - C^{\Lambda_\delta^s}(\{x\}) \subseteq X \backslash A \subseteq U$. Hence $(X, \tau_1, \tau_2)$ is pairwise $\Lambda_\delta^s - R_0$.

**Theorem 5.3** A bitopological space $(X, \tau_1, \tau_2)$ is pairwise $\Lambda_\delta^s - R_1$ if and only if for every pair of points $x$ and $y$ of $X$ such that $ij - C^{\Lambda_\delta^s}(\{x\}) \neq ji - C^{\Lambda_\delta^s}(\{y\})$ , there exists a $ij - \Lambda_\delta^s$ open set $U$ and $ji - \Lambda_\delta^s$ open set $V$ such that $x \in V$ , $y \in U$ and $U \cap V = \phi$.

***Proof.*** Suppose that $(X, \tau_1, \tau_2)$ is pairwise $\Lambda_\delta^s - R_1$. Let $x, y$ be points of $X$ such that $ij - C^{\Lambda_\delta^s}(\{x\}) \neq ji - C^{\Lambda_\delta^s}(\{y\})$. Then there exist a $ij - \Lambda_\delta^s$ open set $U$ and a $ji - \Lambda_\delta^s$ open set $V$ such that $x \in ij - C^{\Lambda_\delta^s}(\{x\}) \subseteq V$ and $y \in ji - C^{\Lambda_\delta^s}(\{y\}) \subseteq U$ . On the other hand, suppose that there exists a $ij - \Lambda_\delta^s$ open set $U$ and $ji - \Lambda_\delta^s$ open set $V$ such that $x \in V$ , $y \in U$ and $U \cap V = \phi$. Since every pairwise $\Lambda_\delta^s - R_1$ space is pairwise $\Lambda_\delta^s - R_0$, $ij - C^{\Lambda_\delta^s}(\{x\}) \subseteq V$ and $ji - C^{\Lambda_\delta^s}(\{y\}) \subseteq U$ . This completes the proof.

**Theorem 5.4** A pairwise $\Lambda_\delta^s - R_0$ space $(X, \tau_1, \tau_2)$ is pairwise $\Lambda_\delta^s - R_1$ if for each pair of points $x$ and $y$ of $X$ such that $ij - C^{\Lambda_\delta^s}(\{x\}) \cap ji - C^{\Lambda_\delta^s}(\{y\}) = \phi$, there exist disjoint sets $U \in ij - \Lambda_\delta^s O(X, \tau_1, \tau_2)$ and $V \in ji - \Lambda_\delta^s O(X, \tau_1, \tau_2)$ such that $x \in U$ and $y \in V$.

***Proof.*** It follows directly from definition 4.1 and theorem 4.7.

**Theorem 5.5** In a bitopological space $(\mathrm{X},\tau_1,\tau_2)$, the following statements are equivalent:

(1) $(\mathrm{X},\tau_1,\tau_2)$ is pairwise $\Lambda_\delta^s - \mathrm{R}_1$.

(2) For any two distinct points $x,y \in X$, $ij - C^{\Lambda_\delta^s}(\{x\}) \neq ji - C^{\Lambda_\delta^s}(\{y\})$ implies that there exist a $ij - \Lambda_\delta^s$ closed set $F_1$ and a $ji - \Lambda_\delta^s$ closed set $F_2$ such that $x \in F_1$, $y \in F_2$, $x \notin F_2$, $y \notin F_1$ and $X = F_1 \cup F_2$.

***Proof.*** (1) $\Longrightarrow$ (2) Suppose that $(\mathrm{X},\tau_1,\tau_2)$ is pairwise $\Lambda_\delta^s - \mathrm{R}_1$. Let $x,y \in X$ such that $ij - C^{\Lambda_\delta^s}(\{x\}) \neq ji - C^{\Lambda_\delta^s}(\{y\})$. By theorem 5.3, there exist disjoint sets $V \in ij - \Lambda_\delta^s(\mathrm{X},\tau_1,\tau_2)$ and $U \in ji - \Lambda_\delta^s(\mathrm{X},\tau_1,\tau_2)$ such that $x \in U$ and $y \in V$. Then $F_1 = X\backslash V$ is a $ij - \Lambda_\delta^s$ closed set and $F_2 = X\backslash U$ is a $ji - \Lambda_\delta^s$ closed set such that $x \in F_1$, $x \notin F_2$, $y \in F_2$, $y \notin F_1$ and $X = F_1 \cup F_2$.

(2) $\Longrightarrow$ (1) Let $x,y \in X$ such that $ij - C^{\Lambda_\delta^s}(\{x\}) \neq ji - C^{\Lambda_\delta^s}(\{y\})$. Hence for any two distinct points $x,y$ of $X$, $ij - C^{\Lambda_\delta^s}(\{x\}) \cap ji - C^{\Lambda_\delta^s}(\{y\}) = \phi$. Then by theorem 4.3, $(\mathrm{X},\tau_1,\tau_2)$ is pairwise $\Lambda_\delta^s - \mathrm{R}_0$. By (2), there exists a $ij - \Lambda_\delta^s$ closed set $F_1$ and a $ji - \Lambda_\delta^s$ closed set $F_2$ such that $X = F_1 \cup F_2$, $x \in F_1$, $y \in F_2$, $x \notin F_2$, $y \notin F_1$. Therefore, $x \in X\backslash F_2 = U \in ji - \Lambda_\delta^s(\mathrm{X},\tau_1,\tau_2)$ and $y \in X\backslash F_1 = V \in ij - \Lambda_\delta^s(\mathrm{X},\tau_1,\tau_2)$ which implies that $ij - C^{\Lambda_\delta^s}(\{x\}) \subseteq U$, $ji - C^{\Lambda_\delta^s}(\{y\}) \subseteq V$ and $U \cap V = \phi$. Hence $(\mathrm{X},\tau_1,\tau_2)$ is pairwise $\Lambda_\delta^s - \mathrm{R}_0$.

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